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China Classification Society

Guidelines on Advanced Buckling Assessment of Hull Structure of Ore Carrier

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Brief Description

China Classification Society (CCS) has newly developed *Guidelines on Advanced Buckling Assessment of Hull Structure of Ore Carrier* to align with the trend toward large-scale ship structures and modernisation of ship structure design. The Guidelines are based on advanced buckling methodology in terms of elasto-plastic/ultimate capacity of stiffened panel structures and apply to the advanced buckling assessment of hull structures of ore carriers.

Three state-of-the-art methods for advanced buckling assessment are incorporated in the Guidelines: the Closed-Form Method (CFM), the Elasto-Plastic Method (EPM) and the Non-Linear Finite Element Method (NLFEM). CFM is based on the buckling requirements having been applied in the IACS (International Association of Classification Societies) *Common Structural Rules for Bulk Carriers and Oil Tankers*, and further supplemented/improved concerning the structural characteristics of large-scale ore carriers. EPM is based on the latest advanced elasto-plastic buckling assessment method developed by CCS and can solve the special plate panel buckling issue. In addition, with the working experiences of CCS in the field of non-linear analysis practices in recent years and the kinds of literature, NLFEM is proposed in the Guidelines with relevant requirements using the Non-Linear Finite Element Method on the ultimate capacity of stiffened plate panels.

If applicable, the Guidelines can also be implemented as a reference for advanced buckling assessment of stiffened panel structures to other ships and marine/offshore structures.

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Chapter 1 General

1.1 General requirements

1.1.1 Based on the advanced buckling analysis methodology for elasto-plastic/ultimate capacity of stiffened panel structures and considering the structural characteristics of ore carriers and direct calculation requirements, the Guidelines provide the advanced buckling assessment methods for stiffened panel for hull structures of ore carriers, which apply to the buckling and ultimate strength assessment of shell envelope, inner hull, inner bottom, deck, bulkhead and web structures of primary and local supporting members, as well as other members including struts, pillars, cross ties and corrugated bulkheads.

1.1.2 Where the Guidelines are applied to the buckling strength assessment for structures of ore carriers, relevant requirements for buckling assessment as specified in CCS *Guidelines for Direct Calculation of Hull Structure Strength of Ore Carrier* are to be considered. In case of any deviation from the *Guidelines for Direct Calculation of Hull Structure Strength of Ore Carrier*, the Guidelines herein prevail.

1.1.3 For each structural member, the characteristic buckling strength is to be taken as corresponding to the most unfavorable/critical buckling failure mode.

1.1.4 In the Guidelines, compressive and shear stresses are to be taken as positive, tension stresses are to be taken as negative.

1.1.5 Unless otherwise specified, the structural member thickness is to be deducted in accordance with 1.6 of this Chapter.

1.1.6 Unless otherwise specified, the calculated stresses are to be corrected in accordance with 1.7 of this Chapter.

1.1.7 Other advanced buckling assessment methods may be acceptable with agreement from CCS.

1.1.8 The Guidelines can also be implemented as a reference for advanced buckling assessment of stiffened panel structures to other ships and marine/offshore structures.

1.2 Advanced buckling method

1.2.1 Buckling/instability is one of the major structural failure modes. According to the classical buckling strength theory, the stability of structures refers to linear elastic buckling. However, local elastic buckling of plate panels in hull structures might not lead to the critical state, i.e. structural collapses. This is because of the inherent structural redundancy associated with the arrangement/configuration of the hull structure, as shown in Figure 1.2.1 (1). After reaching the elastic buckling strength, the structure goes into the elasto-plastic stage (post-buckling/plastic buckling) and can continue to bear additional external loads through stress redistribution until the structure collapses, as shown in Figure 1.2.1 (2). At the time when the structure collapses, the external loads applied to the structure are referred to as the maximum load-carrying capacity of the structure, or the ultimate capacity or ultimate strength of the structure, as shown in Figure 1.2.1 (3).

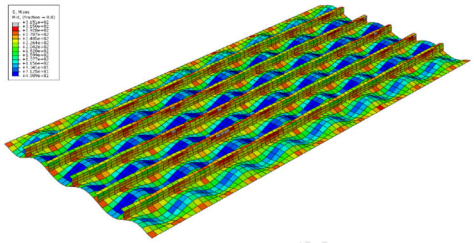


Figure 1.2.1 (1) Linear Elastic Buckling Mode of Stiffened Plate Panel

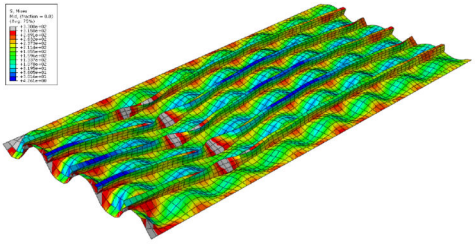


Figure 1.2.1 (2) Collapse Mode of Stiffened Plate Panel

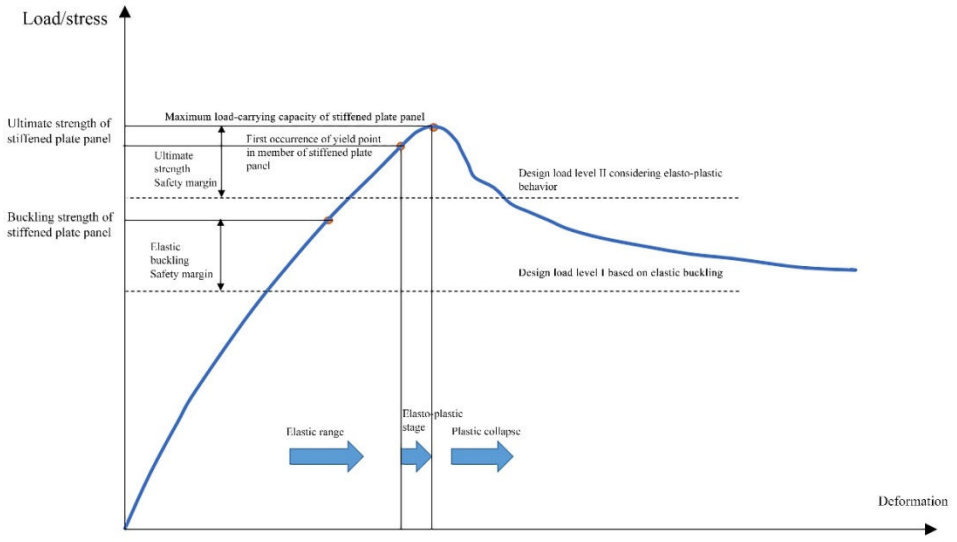


Figure 1.2.1 (3) Stress-deformation Schematic Diagram of Stiffened Plate Panel during Ultimate Capacity Analysis Process

1.2.2 In the Guidelines, advanced buckling methods are primarily defined as non-linear buckling assessment methods that can effectively account for the post-buckling and plastic buckling behaviours of plate panels and the ultimate strength of stiffeners. The purpose of applying the advanced buckling methods is to make further use of the ultimate capacity of the stiffened panel structure while ensuring no collapse of the stiffened plate panels occurs.

1.2.3 For the advanced buckling methods, such effects as non-linear geometrical behaviour, inelastic material behaviour, initial imperfection and residual stresses, as well as the combined action under complex stresses and various boundary conditions are to be taken into account, as shown in Figure 1.2.3.

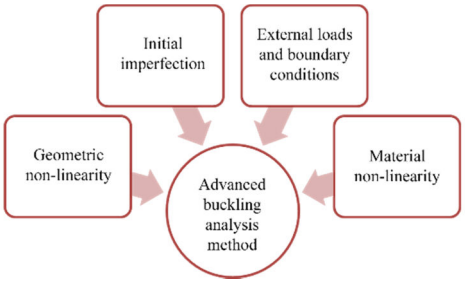


Figure 1.2.3 Relevant Factors for Advanced Buckling Methods

1.2.4 In the Guidelines, the three advanced buckling assessment methods are to be given:

1) Closed-Form Method (CFM) - semi-empirical buckling capacity interaction formula derived by fitting test data and numerical calculation result;

2) Elasto-Plastic Method (EPM) - to solve buckling capacity with the elastic large-deflection shell theory and via rigid-plastic analysis;

3) Non-Linear Finite Element Method (NLFEM) - a finite element numerical analysis method based on non-linear mechanics, to derive the ultimate load and deformation state of plate panel structures by tracking the equilibrium path through incremental iteration and based on the non-linear large deformation theory.

1.2.5 The application of the three advanced buckling methods mentioned in 1.2.4 of this Chapter, and their hierarchy as well as alternatives are described as follows:

(1) CFM is capable of assessing the buckling and ultimate strength for unstiffened/longitudinal stiffened plate panels (including webs in way of openings) under the combined action of bi-axial stress, shear stress and lateral pressure, and can take into account the

stress modes and various boundary conditions linearly distributed at the edge of the plate panel. It is suitable for corresponding design and plan approval, so that is generally preferred for engineering practice;

(2) EPM is currently capable of assessing the buckling and ultimate strength of unstiffened/longitudinal stiffened/orthogonal(bi-axial) stiffened plate panel under the combined action of bi-axial stress, shear stress and lateral pressure, and can be used for structural design and plan approval under the above loads. EPM can serve as an alternative to CFM if unreasonable results are obtained with CFM.

(3) NLFEM is generally capable of calculating the ultimate capacity of various structural arrangements, and in principle can serve as an alternative to CFM and EPM. However, NLFEM is only used to improve the accuracy for special structures and individual cases (if necessary), and the detailed assessment process is to be approved by CCS in advance.

1.3 Application and assessment process

1.3.1 In the Guidelines, buckling assessment is to be carried out in accordance with the following chapters and main workflows (as shown in Figure 1.3.1):

(1) Chapter 1 - general requirements for advanced buckling assessment methods and requirements on allowable buckling utilization factors, buckling acceptance criteria of structural components, thickness deduction and stress corrections;

(2) Chapter 2 - requirements for application of buckling assessment on stiffened and unstiffened panels of hull structures regarding ore carriers, such as shell envelope, inner hull, inner bottom, deck, bulkhead and web structures of primary and local supporting members, as well as other members including struts, pillars, cross ties and corrugated bulkheads, including requirements on plate panel idealization, types of plate panels and applicable methods of assessment, and requirements for the calculation of reference stresses of plate panels;

(3) Chapter 3 - methods for buckling assessment with CFM, including requirements for calculation of buckling and ultimate capacity of stiffened and unstiffened panel structures (also including curved plate panels, webs in way of openings etc.) as well as struts, pillars, cross ties and corrugated bulkheads;

(4) Chapter 4 - methods for buckling assessment with EPM, including requirements for calculation of buckling and ultimate capacity of stiffened and unstiffened panel structures;

(5) Chapter 5 - methods for buckling assessment with NLFEM, including requirements for non-linear finite element computational analysis of the ultimate capacity for stiffened plate panels.

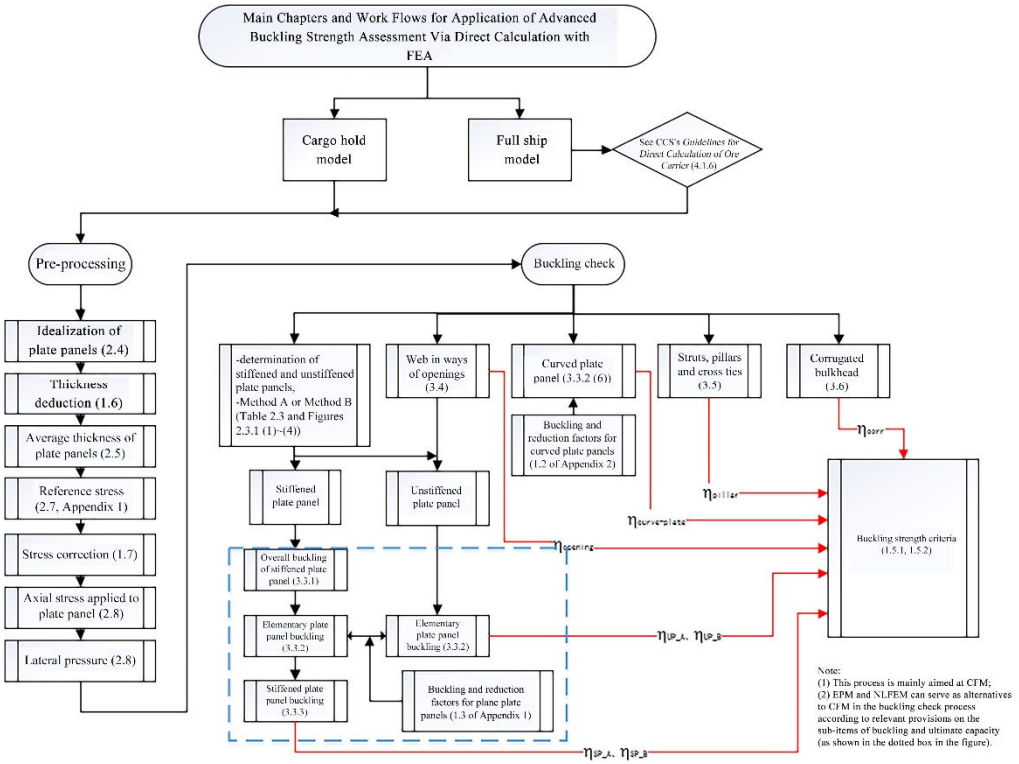


Figure 1.3.1 Main Chapters and Work Flows for Application of Advanced Buckling Strength Assessment Via Direct Calculation with Finite Element Analysis

1.3.2 Unless otherwise specified, the buckling assessment for stiffeners mentioned in the Guidelines applies to stiffened plate panels with stiffeners fitted along the long edge of the plate panel.

1.3.3 Enlarged stiffener structures (with or without web stiffeners) used as permanent means of access (PMA) are to meet the requirements for buckling assessment in the Guidelines. For structures with web stiffeners, generally SP-B method is to be used for buckling assessment; for structures without web stiffeners, generally SP-A method is to be used for buckling assessment by taking enlarged stiffeners as ordinary stiffeners with their attached plating.

1.4 Buckling utilization factor

1.4.1 The buckling utilization factor η is defined as the ratio of the applied load to the corresponding ultimate capacity or buckling strength.

1.4.2 For combined loads, the buckling utilization factor η_{act} is defined as the ratio of the applied equivalent working stress to the corresponding equivalent buckling ultimate capacity, i.e.:

$$\eta_{act} = \frac{W_{act}}{W_u} = \frac{1}{\gamma_c}$$

Where: W_{act} - equivalent working stress, as shown in Figure 1.4.2;

W_u - equivalent buckling stress capacity, as shown in Figure 1.4.2;

γ_c - stress multiplication factor at failure, that is, the structure fails when each stress component (excluding lateral pressure) changes proportionally to γ_c times the actual working stress. For CFM, the detailed application is defined in 3.1 of Chapter 3.

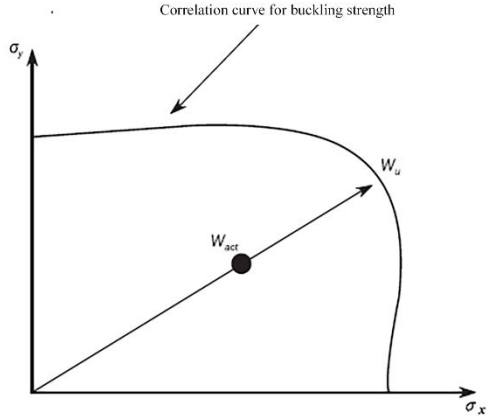


Figure 1.4.2 Example of Buckling Capacity and Buckling Utilization Factor

1.5 Buckling acceptance criteria

1.5.1 The acceptance criteria for the buckling assessment of structures required by the Guidelines are as follows:

$$\eta_{act} \leq \eta_{all}$$

Where: η_{act} - buckling utilization factor corresponding to working stress. The maximum values of the buckling utilization factors corresponding to each failure mode are to be taken for different buckling assessment methods and selected analysis models for unstiffened/stiffened panels, and for structures such as webs in way of openings and corrugated bulkheads. For different buckling assessment methods, the analysis models and failure modes are defined in 3.2.5 of Chapter 3 and 4.2.2 of Chapter 4. For the definition and calculation of the buckling utilization factors corresponding to various failure modes, see 1.4 of this Chapter.

η_{all} *- allowable buckling utilization factor, as shown in Table 1.5.1.*

Allowable Buckling Utilization Factor η_{all} Table 1.5.1

Structural components	η_{all}, allowable buckling utilization factor
Panels and stiffeners Stiffened and unstiffened plate panels Web in ways of openings	1.0

Pillars, struts and cross ties	0.75
Corrugation of vertically corrugated bulkheads with lower stool and horizontally corrugated bulkheads, under lateral pressure from liquid loads, for shell elements only. Supporting structure in way of lower end of corrugated bulkheads without lower stool.	0.9
Corrugation of vertically corrugated bulkheads without lower stool under lateral pressure from liquid loads, for shell elements only.	0.81

Notes: ① Supporting structure for a transverse corrugated bulkhead refers to the structure in longitudinal direction within half a web frame spacing forward and aft of the bulkhead, and within a vertical extent equal to the corrugation depth.

② Supporting structure for a longitudinal corrugated bulkhead refers to the structure in transverse direction within three web frame spacings from each side of the bulkhead, and within a vertical extent equal to the corrugation depth.

1.5.2 Where the calculation load cases are inconsistent with those in CCS *Guidelines for Direct Calculation of Hull Structure Strength of Ore Carrier*, the allowable buckling utilization factors in Table 1.5.1 need to be considered separately and agreed upon by CCS.

1.6 Thickness deduction

1.6.1 For buckling calculation, the buckling capacity of all plate panels and stiffeners is to be calculated based on the scantling obtained by removing the standard deduction thickness t_r from the gross scantling. See Table 1.6.1 for the standard deduction thickness t_r.

Standard deduction thickness Table 1.6.1

Position		Standard deduction thickness, mm
Within 1.5m below the top of ballast tank	One side exposure to ballast water	1.0
	Both sides exposure to ballast water	2.0
Other components		1.0

1.6.2 If thickness is reduced based on other standard deduction thicknesses or requirements on corrosion additions, the allowable buckling utilization factor η_{all} in 1.5.1 of this Chapter is to be considered separately and agreed upon by CCS.

1.7 Correction of calculated stresses

1.7.1 Unless otherwise specified, where the buckling assessment is performed in accordance with the Guidelines, the stresses (including components of normal stress and shear stress) calculated by the finite element model based on the as-built thickness (regardless of the ship owner's additional thickness) are to be corrected as follows:

$$\sigma_A = \frac{t}{t - t_r}\sigma \qquad \text{N/mm}^2$$

Where: σ_A - corrected buckling working stress (including components of normal stress and she stress), N/mm^2;

σ - stresses (including normal stress and shear stress) calculated with the finite element method, N/mm^2;

t – as-built thickness, mm;

t_r - standard deduction thickness, mm, as shown in Table 1.6.1.

Note: If the working stress is only caused by overall bending, such as the hull girder longitudinal bending stress, it is not necessary to correct the calculated stress corresponding to the buckling assessment under such bending load, but subject to agreement with CCS.

Chapter 2 Application of Buckling Assessment

2.1 General requirements

2.1.1 Concerning the stiffened and unstiffened panel for hull structures of ore carrier, such as shell envelope, inner hull, inner bottom, deck, bulkhead and web structures of primary and local supporting members, as well as other members including struts and corrugated bulkheads, this chapter provides requirements for application of the buckling assessment method, including the requirements of plate panel idealization, types of plate panels, applicable assessment methods and calculation of reference stresses in plate panels. In addition, criteria for buckling checks by means of finite element method are given.

2.1.2 Buckling assessment with FEA is carried out for the following hull structures of ore carriers:

(1) Stiffened plate panels and unstiffened plate panels (including curved plate panels);

(2) Web plate panels in way of openings;

(3) (Compressed) struts and pillars (if applicable);

(4) Cross ties;

(5) Corrugated bulkheads.

2.1.3 For the buckling checks for direct calculation on hull structure strength of ore carrier, the working stress in buckling assessment is to be obtained by correcting the stresses obtained from FEA on cargo hold and/or full ship model according to relevant requirements in CCS *Guidelines for Direct Calculation of Hull Structure Strength of Ore Carrier*. The lateral pressure on the plate panel is taken as the corresponding lateral load applied to the finite element model.

2.1.4 Before buckling strength checking, the associated pre-processing work is required to be performed, such as plate panel idealization, thickness deduction, plate panel thickness averaging, working stress correction and conversion of corrected working stress into reference stress, etc.

2.1.5 For those not covered in this chapter, they are subject to relevant requirements in CCS *Guidelines for Direct Calculation of Hull Structure Strength of Ore Carrier*.

2.2 Assessment method

2.2.1 The buckling assessment is carried out according to one of the two methods taking into account different boundary condition types:

(1) Method A: All the edges of the elementary plate panel are forced to remain straight (but free to move in the in-plane directions) due to the surrounding structure/neighboring plates. This method is suitable for large-area continuous plate panels, such as bottom plating, side shell plating, inner hull, inner bottom, deck, longitudinal and transverse bulkhead.

(2) Method B: The edges of the elementary plate panel are not forced to remain straight due to low in-plane stiffness at the edges and/or no surrounding structure/neighboring plates. This method is suitable for the webs of some primary supporting members subject to weak restraint in panels.

Note: Generally, Method B has a buckling capacity less than or equal to that of Method A, and a linear elastic buckling stress greater than or equal to that of Method A.

2.2.2 For the detailed application of "Method A" and "Method B", see 2.3.1 of this chapter.

2.3 Types of plate panels structure of ore carrier and applied assessment methods

2.3.1 For each of the ore carrier structural members, its applicable types of stiffened plate panels/unstiffened plate panels and applicable methods for assessment (Method A/Method B) are determined according torelated structural arrangement, stiffener arrangement and location of the plate panel. As typically described in Table 2.3.1 and Figure 2.3.1 (1) ~ Figure 2.3.1 (4), one of the following four models, as a combination of applicable plate panel models and assessment methods, are usually to be applied for buckling assessment:

(1) **SP-A** - stiffened plate panel, assessed with method A;

(2) **SP-B** - stiffened plate panel, assessed with method B;

(3) **UP-A** - unstiffened plate panel, assessed with method A;

(4) **UP-B** - unstiffened plate panel, assessed with method B.

Types of Plate Panels and Corresponding Assessment Methods Table 2.3.1

Structural elements	**Assessment method**	**Normal plate panel definition**
Longitudinal structure, see Figure 2.3.1 (1)		
Longitudinal stiffened plate panel Shell envelope Deck Inner hull Hopper tank side	SP-A	Length: between web frames Width: between primary supporting members (PSM)

Longitudinal bulkheads		
Double bottom longitudinal girders in line with longitudinal bulkhead or connected to hopper tank side	SP-A	Length: between web frames Width: full web depth
Web of double bottom longitudinal girders not in line with longitudinal bulkhead or not connected to hopper tank side	SP-B	Length: between web frames Width: full web depth
Longitudinal web in double side space connected to hopper tank side	SP-A	Length: between web frames Width: full web depth
Longitudinal web in double side space not connected to hopper tank side	SP-B	Length: between web frames Width: full web depth
Web of single skin longitudinal girders or stringers	UP-B	Plate between local stiffeners/face plate/PSM
Hatch coaming	SP-B	Plate between local stiffeners/top plate/PSM
Transverse structure, see Figure 2.3.1 (2)		
Web of transverse deck frames including brackets	SP-A	Plate between local stiffeners/face plate/PSM
Transverse web in double side space	SP-A	Length: full web depth Width: between primary supporting members
Irregularly stiffened plate panels, e.g. web plate panels in way of bottom side tank and bilge	UP-B	Plate between local stiffeners/face plate/PSM
Double bottom floors	SP-A	Length: full web depth Width: between primary supporting members
Vertical girders including brackets	SP-A	Plate between vertical girder stiffeners/face plate/PSM
Cross tie web	SP-A	Plate between vertical girder stiffeners/face plate/PSM
Transverse bulkhead, see Figure 2.3.1 (3)		
Regularly stiffened bulkhead plate panels, including the secondary buckling stiffeners perpendicular to the regular stiffeners (such as carlings)	SP-A	Length: between primary supporting members Width: between primary supporting members
Irregularly stiffened bulkhead plate panels, e.g. web plate panels in way of bottom side tank and bilge	UP-B	Plate between local stiffeners/face plate
Web of horizontal bulkhead girders including brackets	UP-A	Plate between local stiffeners/face plate
Upper/lower stool including stiffeners	SP-A	Length: between internal web diaphragms

		Width: length of stool side
Stool internal web diaphragm	UP-B	Plate between local stiffeners/face plate/PSM
Cross deck, see Figure 2.3.1 (4)		
Cross deck	SP-A	Plate between local stiffeners/PSM
Notes: ① SP stands for stiffened plate panel; ② UP stands for unstiffened plate panel; ③ A stands for Method A; ④ B stands for Method B;		

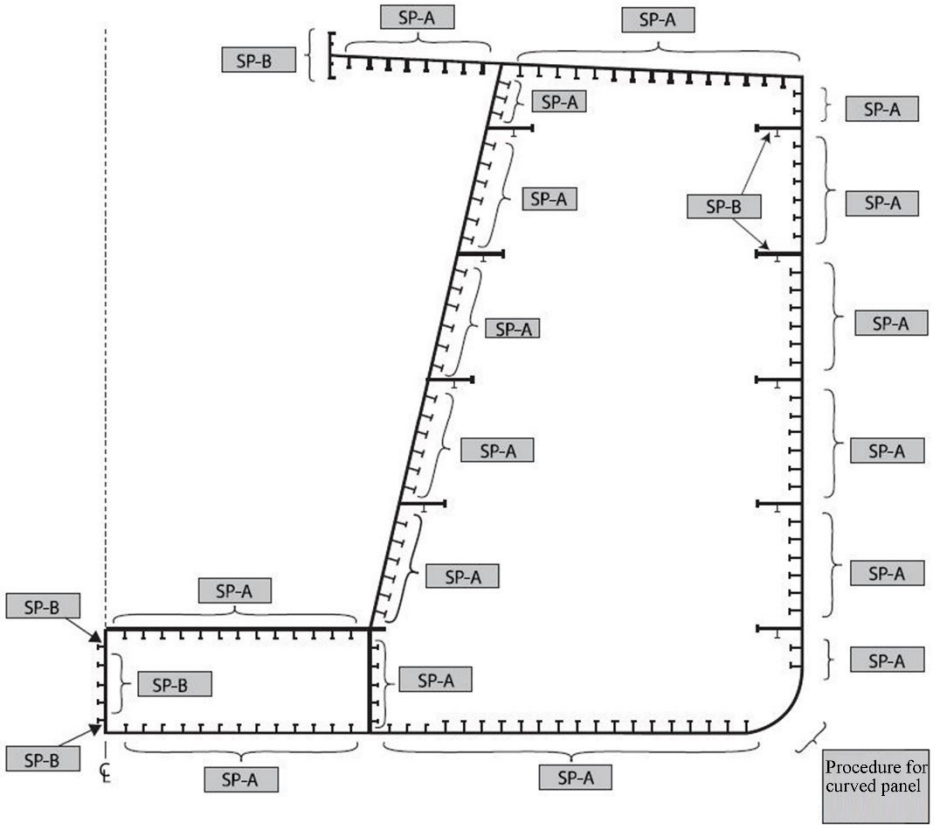


Figure 2.3.1 (1) Longitudinal Plate Panel of Typical Ore Carrier

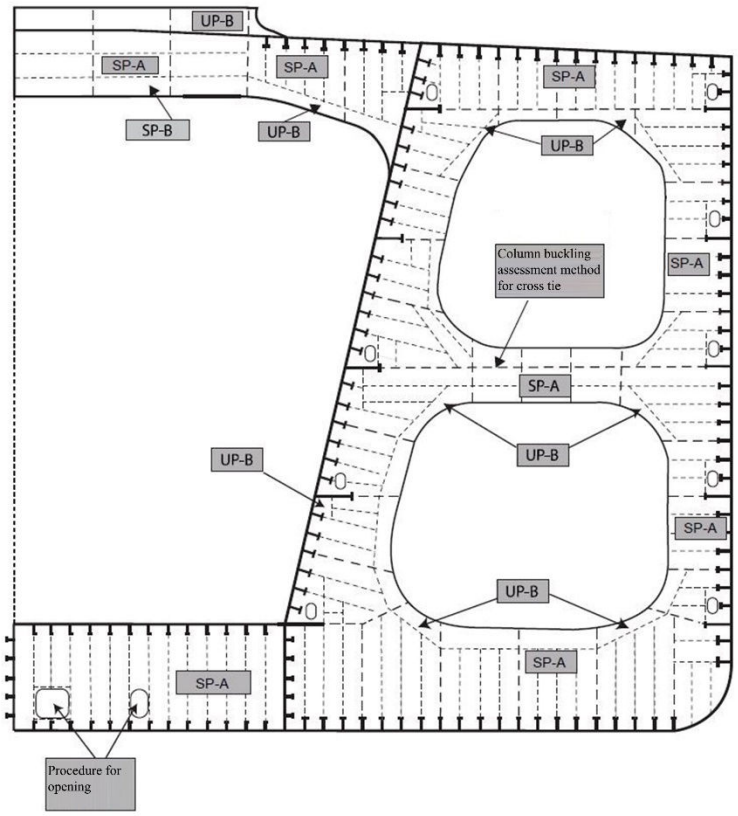


Figure 2.3.1 (2) Transverse Web Frame of Typical Ore Carrier

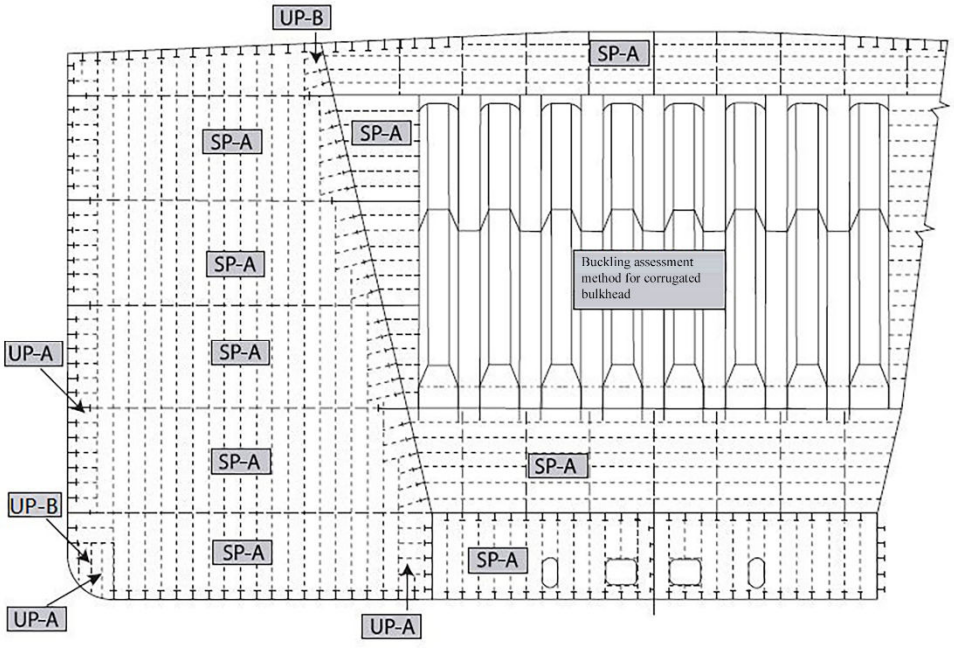


Figure 2.3.1 (3) Transverse Bulkhead of Typical Ore Carrier (Corrugated Bulkhead)

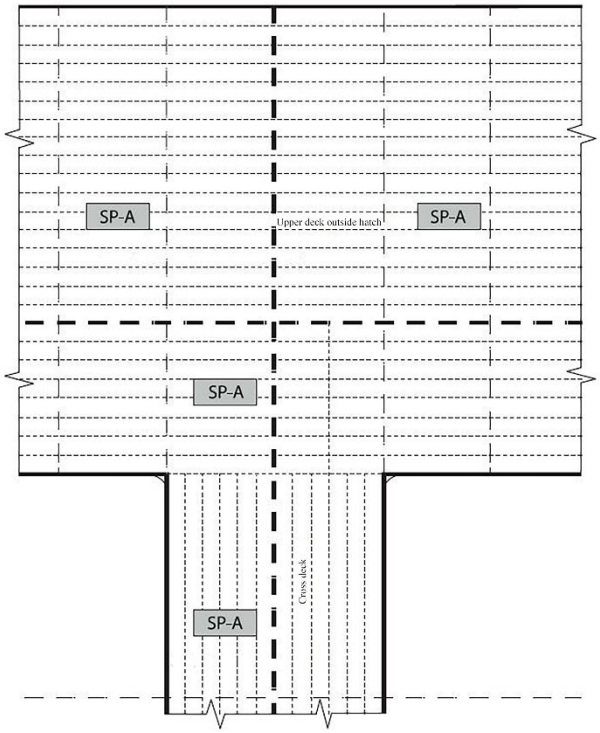


Figure 2.3.1 (4) Cross Deck and Outer Deck

2.4 Modeling of stiffened and unstiffened plate panels

2.4.1 Model of stiffened plate panel

(1) Stiffeners with attached plating are to be simulated as stiffened plate panels. See Table 2.3.1 for the definition of plate panel.

(2) If the stiffener properties or stiffener spacing varies within the stiffened panel, the calculations are to be performed separately for all configurations of the elementary plate panel, i.e. for each stiffener and plate between the stiffeners. Plate thickness, stiffener properties and stiffener spacing of the elementary plate panel at the considered location are to be assumed for the whole panel.

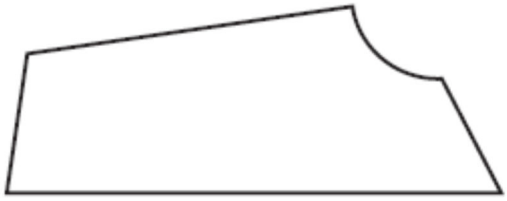
2.4.2 Model of unstiffened plate panel

In way of web frames, stringers and brackets, the geometry of the plate panel (i.e. plate bounded by web stiffeners/face plate) may not have a rectangular shape. In this case, an equivalent rectangular plate panel is to be defined according to (1) for irregular geometry and (2) for triangular geometry and to comply with buckling assessment.

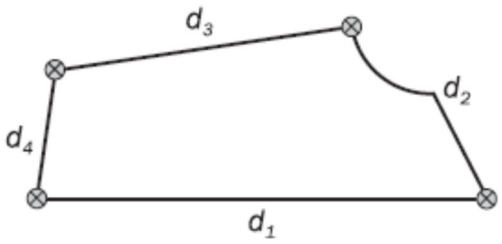
(1) Idealization for model of plate panel with irregular geometry

Unstiffened plate panels with irregular geometry are to be idealized to equivalent plate panels ($a \times b$) according to the following procedure:

① The four corners closest to a right angle (90°) in the bounding polygon for the plate panel are identified;

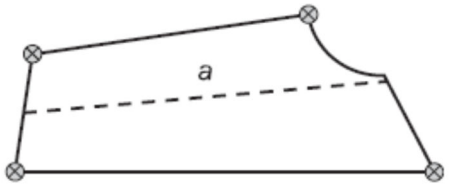


② The distances along the plate panel bounding polygon between the corners are calculated, i.e. the sum of all the straight line segments between the end points;



③ The pair of opposite edges with the smallest total length is identified, i.e. minimum of $d_1 + d_3$ and $d_2 + d_4$;

④ A line joins the middle points of the chosen opposite edges (i.e. a mid point is defined as the point at half the distance from one end). This line defines the longitudinal direction for the capacity model. The length of the line defines the length *a* of the capacity model.



⑤ The length of shorter side, b in mm, is to be taken as:

$$b = A / a$$

Where: A - area of the plate, in mm^2;

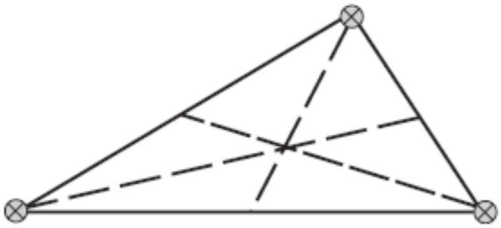
a - length defined in ④, in mm;

⑥ The stresses from the direct strength analysis are to be transformed into the local coordinate system of the equivalent rectangular plate panel. These stresses are to be used for the buckling assessment.

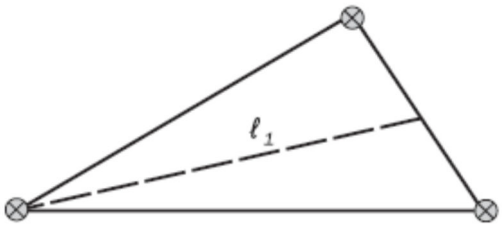
(2) Modeling of plate panel with triangular geometry

Unstiffened plate panels with triangular geometry are to be idealized to equivalent plate panels ($a \times b$) for plate buckling assessment according to the following procedure:

① Medians are constructed as shown below;



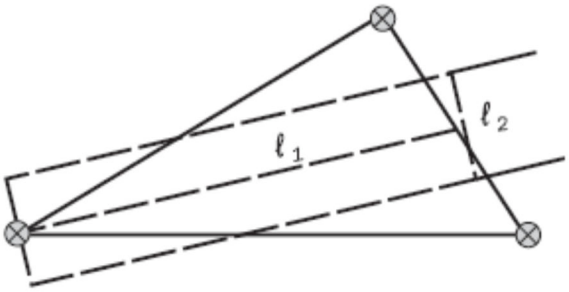
② The longest median is identified. This median the length of which is l_1, in mm, defines the longitudinal direction for the capacity model;



③ The width of the model, l_2, in mm, is to be taken as:

$$l_2 = A / l_1$$

Where: A - area of the plate, in mm^2;



④ The lengths of shorter side, *b*, and of the longer side, *a*, in mm, of the equivalent rectangular plate panel are to be taken as:

$$b = \frac{l_2}{C_{tri}}$$

$$a = l_1 C_{tri}$$

Where:

$$C_{tri} = 0.4\frac{l_2}{l_1} + 0.6$$

⑤ The stresses from the direct strength analysis are to be transformed into the local coordinate system of the equivalent rectangular plate panel. These stresses are to be used for the buckling assessment.

2.5 Average thickness of plate panel

2.5.1 Where the plate thickness along a plate panel is not constant, the following weighted mean plate thickness is to be used in the buckling assessment:

$$t_{avr} = \frac{\sum_{i=1}^{n} A_i t_i}{\sum_{i=1}^{n} A_i} \text{ mm}$$

Where: A_i - area of the i-th plate element, mm^2;

t_i - thickness of the i-th plate element, mm;

n - number of finite elements within the buckling plate panel.

2.6 Yield stress of plate panel

2.6.1 The plate panel yield stress R_{eH_P} is taken as the minimum value of the specified yield stresses of the elements within the plate panel.

2.7 Reference stress

2.7.1 The stress distribution is to be taken from the direct strength analysis and applied to the buckling calculation model with the actual stresses corrected in accordance with 1.7 of Chapter 1.

2.7.2 With CFM, reference stresses of plate panel boundary are calculated according to Appendix 1 of this chapter. For the other two buckling assessment methods, it is calculated with the area-weighted average stress method.

2.8 Lateral pressure

2.8.1 For plate panel buckling assessment, the lateral pressures applied to elements in direct calculation and analysis are to be considered together with other loads under the same calculation conditions.

2.8. Where the lateral pressure is not constant over a buckling plate panel defined by a number of finite plate elements, an average lateral pressure, P_{avr}, is calculated using the following formula:

$$P_{avr} = \frac{\sum_{i=1}^{n} A_i P_i}{\sum_{i=1}^{n} A_i} \text{ N/mm}^2$$

Where: A_i - area of the i-th plate element, mm^2;

P_i - lateral pressure of the i-th plate element, N/mm^2;

n - number of elements in the buckling plate panel.

2.9 Corrugated bulkhead

2.9.1 Buckling failure mode

Three buckling failure modes are to be assessed on corrugated bulkheads:

① Corrugation overall column buckling. See Table 2.9.1 for application;

② Corrugation face plate buckling. Buckling assessment is to be carried out for each corrugation face plate;

③ Corrugation web buckling. Buckling assessment is to be carried out for each corrugation web.

Application of overall column buckling for corrugated bulkhead Table 2.9.1

	Corrugation orientation	
	Horizontal	Vertical
Longitudinal bulkheads	Required	Required, when subjected to local vertical forces (e.g. crane loads)
Transverse bulkhead	Required	

2.9.2 Reference stress

The reference stresses for buckling assessment of corrugated bulkheads are to be calculated and taken in accordance with the followings:

(1) The membrane stresses at element centroid are to be used, with the actual stresses corrected in accordance with 1.7 of Chapter 1 for buckling assessment;

(2) The maximum normal stress parallel to the corrugation for local buckling of flange or web is the maximum of the following two stresses:

① The normal stress parallel to the corrugation taken at b/2 from the corrugation ends of the corrugated bulkhead, with b standing for the width of the flange or web;

When the corrugation end is fitted with a shedder plate, the normal stress parallel to the corrugation at end is to be taken at b/2 from the intersection of the shedder plate with the point at mid breadth of the flange or of the web. See Figure 2.9.2 (1) and Figure 2.9.2 (2) for examples of some typical structures.

Note: Due to relatively large stress gradient in the area close to the corrugation end, the stress at b/2, instead of local high stress, is taken for buckling check.

② The normal stress parallel to the corrugation within the mid span of the corrugation.

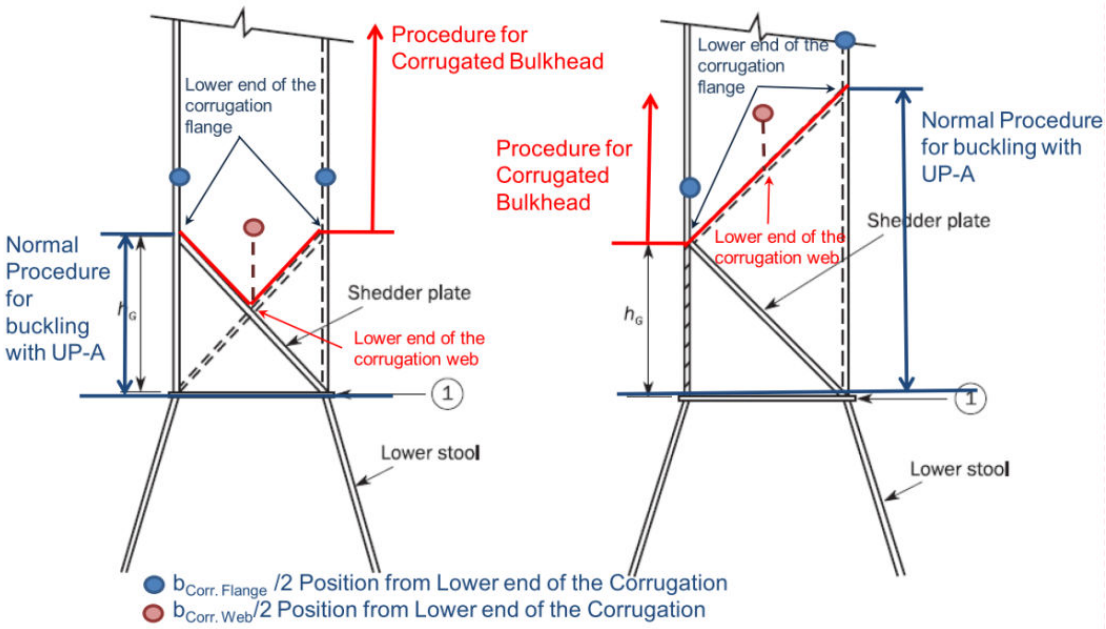


Figure 2.9.2 (1) Definition of Position b/2 from End of Corrugated Bulkhead

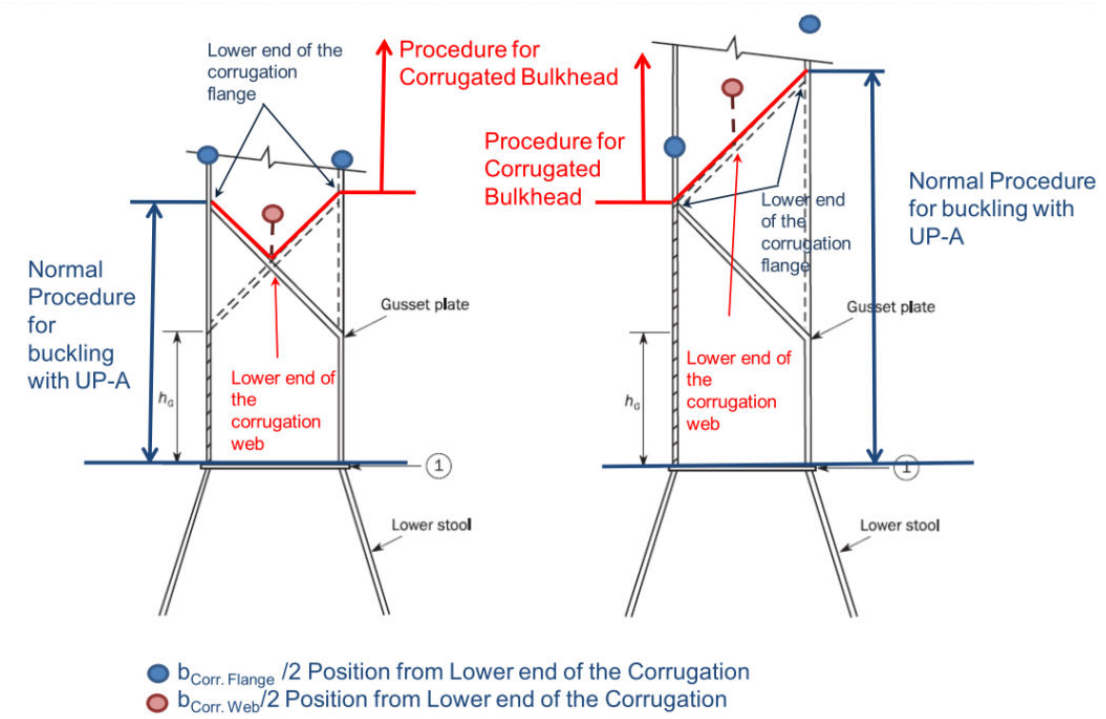


Figure 2.9.2 (2) Definition of Position b/2 from End of Corrugated Bulkhead

(3) The maximum shear stress for local buckling at the flange or web is the shear stress which is maximum at the corrugation flange or web at the point b/2 from ends or within the mid span of the corrugation as shown in Figure 2.9.2 (1) and Figure 2.9.2 (2).

(4) If the corrugation flange or web is divided into multiple elements at width direction, the stress σ_x, σ_y and shear stress τ at the calculation points referred to in (2) and (3) above are to be averaged over width at considered location.

(5) When the stress value at b/2 from ends cannot be obtained directly from elements as shown in Figure 2.9.2 (1) and Figure 2.9.2 (2), the stress components σ_x and σ_y are to be obtained by interpolation of element stresses within 3b range at the corrugation end, with interpolation formulate shown in 1.3.1 of Appendix 1; the stress component τ is to be obtained by linear interpolation of the shear stress of the element nearest to the position b/2 from the end.

(6) If the flange or web contains different plating thicknesses, for each area corresponding to a thickness, the buckling assessment is to be carried out with the necessary maximum stress obtained in (2) and (3) above.

Appendix 1 Stress Based Reference Stresses

1.1 Symbols

1.1.1 Unless otherwise specified, the following definitions of symbols are applicable to this Appendix:

a - length, in mm, of the longer side of the plate panel as defined in 3.1.1 of Chapter 3.

b - length, in mm, of the shorter side of the plate panel as defined in 3.1.1 of Chapter 3.

A_i - area, in mm^2, of the *i*-th plate element of the buckling plate panel.

n - number of plate elements in the buckling plate panel.

σ_{xi} - stress, in N/mm^2, at the centroid of the *i*-th plate element in *x* direction.

σ_{yi} - stress, in N/mm^2, at the centroid of the *i*-th plate element in *y* direction.

ψ - edge stress ratio as defined in 3.1.1 of Chapter 3.

τ_i - shear stress, in N/mm^2, at the centroid of the *i*-th plate element of the buckling plate panel.

1.2 General

1.2.1 Stress based method

(1) This Appendix provides a method to determine stresses along edges of the buckling plate panel by second order polynomial distribution, by linear least square method and by area weighted average method. This method is called stress based method. The reference stress is the stress components at centroid of plate element transferred into the local system of the considered buckling plate panel.

(2) Definition

A regular plate panel is the plate panel of rectangular shape;

An irregular plate panel is the plate panel which is not regular, as detailed in 2.4.2 of Chapter 2.

(3) Selection of calculation method for reference stress

① Regular plate panel

The reference stresses are to be taken as defined in 1.3.1 of the Appendix for a regular plate panel when the following conditions are satisfied:

(a) At least, one plate element centroid is located in each third part of the longer side *a* of a regular plate panel;

(b) This element centroid is located at a distance in the local x direction of the plate panel not less than *a/4* to at least one of the element centroids in the adjacent third part of the plate panel;

Otherwise, the reference stresses are to be taken as defined in 1.3.2 of the Appendix for an irregular plate panel.

② Irregular plate panel and curved plate panel

The reference stresses of an irregular plate panel or of a curved plate panel are to be taken as defined in 1.3.2 of the Appendix.

1.3 Reference stress

1.3.1 Regular plate panel

(1) Longitudinal stress for buckling assessment of plate panel

① Longitudinal stress σ_x

The longitudinal stress σ_x applied on the shorter side of the buckling plate panel is to be calculated as follows:

For plate buckling checks, the distribution of $\sigma_x(x)$ is assumed as second order polynomial curve as:

$$\sigma_x = Cx^2 + Dx + E$$

The best fitting curve of stress $\sigma_x(x)$ is to be obtained by minimizing the area weighted square error Π considering the area of each element as a weighting factor:

$$\Pi = \sum_{i=1}^{n} A_i \left[\sigma_{xi} - (Cx^2 + Dx + E) \right]^2$$

The unknown coefficients C, D and E should yield zero first partial derivatives, Π with respect to C, D and E respectively.

$$\frac{\partial \Pi}{\partial C} = 2\sum_{i=1}^{n} A_i x_i^2 \left[\sigma_{xi} - (Cx_i^2 + Dx_i + E) \right] = 0$$

$$\frac{\partial \Pi}{\partial D} = 2\sum_{i=1}^{n} A_i x_i \left[\sigma_{xi} - (Cx_i^2 + Dx_i + E) \right] = 0$$

$$\frac{\partial \Pi}{\partial E} = 2\sum_{i=1}^{n} A_i \left[\sigma_{xi} - (Cx_i^2 + Dx_i + E) \right] = 0$$

The unknown coefficients C, D and E can be obtained by solving the above equations.

$$\sigma_{x1} = \frac{1}{b}\int_0^b \sigma_x(x)dx = \frac{b^3}{3} C + \frac{b}{2} D + E$$

$$\sigma_{x2} = \frac{1}{b}\int_{a-b}^{a} \sigma_x(x)dx = (a^2 - ab + \frac{b^3}{3})C + (a - \frac{b}{2})D + E$$

If $-D/2C < b/2$ or $-D/2C > a - b/2$, σ_{x3} is to be ignored. Otherwise, σ_{x3} is taken as:

$$\sigma_{x3} = \frac{1}{b}\int_{x_{\min}}^{x_{\max}} \sigma_x(x)dx = \frac{b^2}{12} C - \frac{D^2}{4C} + E$$

Where:

$$x_{\min} = -\frac{b}{2} - \frac{D}{2C}$$

$$x_{\max} = \frac{b}{2} - \frac{D}{2C}$$

The longitudinal stress is to be taken as:

$$\sigma_x = \max(\sigma_{x1}, \sigma_{x2}, \sigma_{x3})$$

The edge stress ratio is to be taken as:

$$\psi_x = 1$$

(2) For stiffener buckling assessment, the stress $\sigma_x(x)$ applied on the shorter side of the attached plate is to be taken as:

$$\sigma_x = \frac{\sum_1^n A_i \sigma_{xi}}{\sum_1^n A_i}$$

The edge stress ratio for the stress σ_x is equal to 1.0.

(3) Transverse stress σ_y

The transverse stress σ_y applied along the longer side of the buckling plate panel is to be calculated by interpolation of the transverse stresses of all elements up to the shorter side of the considered buckling plate panel.

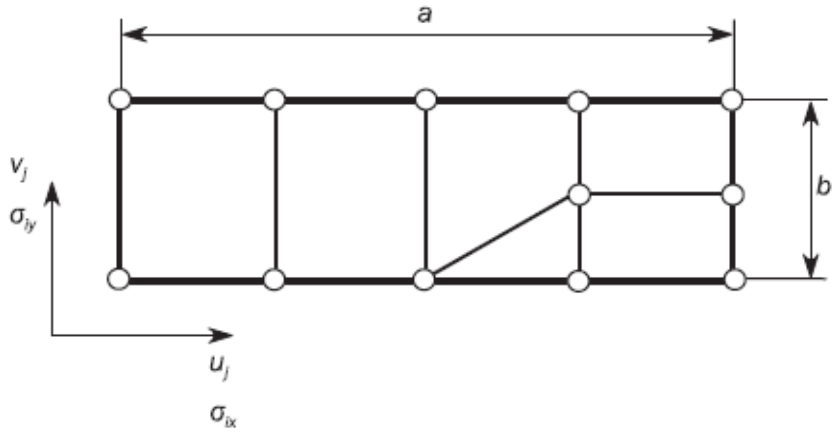


Figure 1.3.1 Buckling Plate Panel

The distribution of $\sigma_y(x)$ is assumed as straight line. Therefore:

$$\sigma_y(x) = A + Bx$$

The best fitting curve of $\sigma_y(x)$ is to be obtained by the least square method, and by minimizing the area weighted square error Π considering the area of each element as a weighting factor:

$$\Pi = \sum_{i=1}^{n} A_i \left[\sigma_{yi} - (A + Bx_i)\right]^2$$

The unknown coefficients A and B should yield zero first partial derivatives, Π with respect to A and B, respectively.

$$\frac{\partial \Pi}{\partial A} = 2\sum_{i=1}^{n} A_i \left[\sigma_{yi} - (A + Bx_i) \right] = 0$$

$$\frac{\partial \Pi}{\partial B} = 2\sum_{i=1}^{n} A_i x_i \left[\sigma_{yi} - (A + Bx_i) \right] = 0$$

The unknown coefficients A and B are obtained by solving the two above equations and are given as follow:

$$\begin{cases} A = \dfrac{\left(\sum_{i=1}^{n} A_i \sigma_{iy}\right)\left(\sum_{i=1}^{n} A_i x_i^2\right) - \left(\sum_{i=1}^{n} A_i x_i\right)\left(\sum_{i=1}^{n} A_i x_i \sigma_{iy}\right)}{\left(\sum_{i=1}^{n} A_i\right)\left(\sum_{i=1}^{n} A_i x_i^2\right) - \left(\sum_{i=1}^{n} A_i x_i\right)^2} \\ B = \dfrac{\left(\sum_{i=1}^{n} A_i\right)\left(\sum_{i=1}^{n} A_i x_i \sigma_{iy}\right) - \left(\sum_{i=1}^{n} A_i x_i\right)\left(\sum_{i=1}^{n} A_i \sigma_{iy}\right)}{\left(\sum_{i=1}^{n} A_i\right)\left(\sum_{i=1}^{n} A_i x_i^2\right) - \left(\sum_{i=1}^{n} A_i x_i\right)^2} \end{cases}$$

$$\sigma_y = \max(A, A + Ba)$$

The edge stress ratio ψ_y for the stress σ_y is taken as:

If $\sigma_y \geq 0$, $\psi_y = \dfrac{\min(A, A + Ba)}{\max(A, A + Ba)}$

If $\sigma_y < 0$, $\psi_y = 1$

(4) Shear stress τ

The shear stress τ is to be calculated using the area weighted average method, and is to be taken as:

$$\tau = \frac{\sum_{i=1}^{n} A_i \tau_i}{\sum_{i=1}^{n} A_i}$$

1.3.2 Irregular plate panel and curved plate panel

(1) Reference stress

The longitudinal, transverse and shear stresses are to be calculated using the area weighted average method. They are to be taken as:

$$\sigma_x = \frac{\sum_{i=1}^{n} A_i \sigma_{xi}}{\sum_{i=1}^{n} A_i}$$

$$\sigma_y = \frac{\sum_{i=1}^{n} A_i \sigma_{yi}}{\sum_{i=1}^{n} A_i}$$

$$\tau = \frac{\sum_{i=1}^{n} A_i \tau_i}{\sum_{i=1}^{n} A_i}$$

The edge stress ratio is to be taken as:

$$\psi_x = 1$$

$$\psi_y = 1$$

Chapter 3 Closed-Form Method (CFM) for Buckling/Ultimate Strength of Stiffened Plate Panels

3.1 Symbols

3.1.1 Unless otherwise specified, the following definitions of symbols and specifications are applicable to this Chapter:

A_s - Sectional area of the stiffener without attached plating, in mm^2.

a- Length of the long side of the plate panel, in mm, as shown in Table 1.2.1 of Appendix 2.

b- Length of the short side of the plate panel, in mm, as shown in Table 1.2.1 of Appendix 2.

d- Length of the side parallel to the axis of the cylinder corresponding to the curved plate panel as shown in Table 1.3.1 of Appendix 2, in mm.

b_{eff} - Effective width of the attached plating of a stiffener, in mm, as defined in 3.3.3(5) of this Chapter.

b_{eff1} - Effective width of the attached plating of a stiffener, in mm, without the shear lag effect, to be taken as:

$$b_{eff1} = \begin{cases} C_x b & \textbf{when} \quad \sigma_x > 0 \\ b & \textbf{when} \quad \sigma_x \le 0 \end{cases}$$

C_x - Reduction factor of the plate panel defined in Table 1.2.1 of Appendix 2 according to case ①.

b_f - Breadth of the stiffener flange, in mm.

b_1、b_2 - Width of plate panel on each side of the stiffener, in mm.

e_f - Distance from attached plating to centre of flange, in mm, as shown in Figure 3.1.1, to be taken as:

$$e_f = \begin{cases} h_w & For\ flat\ bar \\ h_w - 0.5t_f & For\ bulb\ profile \\ h_w + 0.5t_f & For\ angle\ profile,\ L2\ and\ Z\ profiles \\ h_w - d_e - 0.5t_f & For\ L3 \end{cases}$$

F_{long} - Coefficient defined in 3.3.2(4) of this Chapter.

F_{tran} - Coefficient defined in 3.3.2(5) of this Chapter.

h_w - Depth of stiffener web, in mm, as shown in Figure 3.1.1.

l_s - Span, in mm, of stiffener equal to spacing between primary supporting members.

R - Radius of curved plate panel, in mm.

R_{eH_P} - Yield stress of plate, in N/mm^2.

R_{eH_S} - Yield stress of stiffener, in N/mm^2.

E- Young's modulus, in N/mm^2, for steel, $E = 2.06 \times 10^5$.

v- Poisson's ratio, for steel, $\nu = 0.3$.

S- Partial safety factor to be taken as:

S=1.1 for structures which are exposed to local concentrated loads (e.g. container loads on hatch covers, foundations);

S =1.0 for all other cases.

t_p- Thickness of plate panel, in mm.

t_w- Web thickness of stiffener, in mm.

t_f - Flange thickness of stiffener, in mm.

x axis- Local axis of a rectangular buckling plate panel parallel to its long side

y axis- Local axis of a rectangular buckling plate panel perpendicular to its long side

α- Aspect ratio of the plate panel, defined in Table 1.2.1 of Appendix 2 to be taken as:

$$\alpha = \frac{a}{b}$$

β- Coefficient taken as:

$$\beta = \frac{1-\psi}{\alpha}$$

ω - Coefficient taken as:

$$\omega = \min(3, \alpha)$$

σ_x - Stress applied on the edge along x axis of the plate panel, in N/mm^2.

σ_y - Stress applied on the edge along y axis of the plate panel, in N/mm^2.

σ_1 - Maximum stress, in N/mm^2, as shown in Table 1.2.1 of Appendix 2.

σ_2 - Minimum stress, in N/mm^2, $\sigma_2 = \psi\sigma_1$, as shown in Table 1.2.1 of Appendix 2.

σ_E - Elastic buckling reference stress, in N/mm^2, to be defined as:

(1) For the application of plate panel limit state according to 3.3.2(1) of this Chapter:

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_p}{b}\right)^2$$

(2) For the application of limit state of curved plate panel according to 3.3.2(6) of this Chapter:

$$\sigma_E = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_p}{d}\right)^2$$

τ - Applied shear stress, in N/mm^2.

τ_c - Buckling strength in shear, in N/mm^2, as defined in 3.3.2(3) of this Chapter.

ψ- Edge stress ratio, to be defined as:

$$\psi = \frac{\sigma_2}{\sigma_1}$$

γ- Stress multiplier factor acting on loads. When the factor is such that the loads reach the interaction formula, $\gamma = \gamma_c$.

γ_c - Stress multiplier factor at failure.

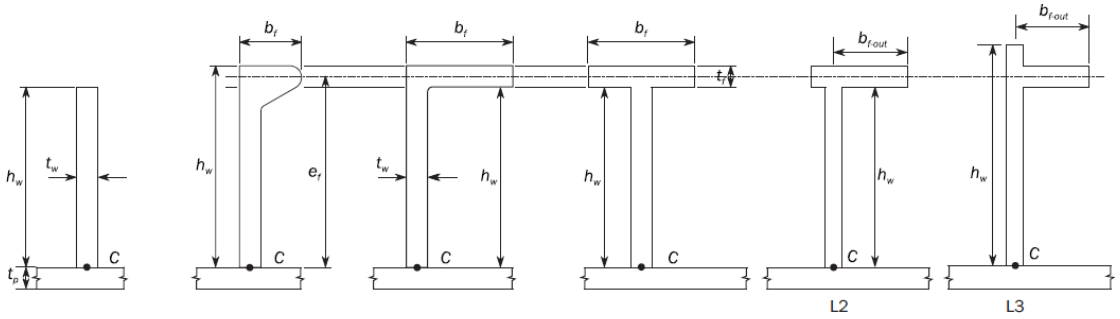


Figure 3.1.1 Stiffener Cross Sectional Scantling Parameters

3.2 General requirements

3.2.1 This section contains the methods for determination of the buckling capacity of plate panels, stiffeners, primary supporting members, struts, pillars and corrugated bulkheads.

3.2.2 For the application of this Chapter, the stresses σ_x, σ_y and τ applied on the structural members are defined in 2.7 of Chapter 2.

3.2.3 The ultimate buckling capacity is calculated by applying the actual stress combination and then increasing or decreasing the stresses proportionally until the interaction formulae defined in 3.3.1, 3.3.2 and 3.3.3(4) are equal to 1.0. During this process, the lateral pressure is to be kept constant.

3.2.4 The buckling utilization factor of the structural member is equal to the maximum utilization factor obtained for the different buckling modes.

3.2.5 The idealized analytical models for the different hull members and their various failure modes to be considered when applying the CFM are given in Table 3.2.5(1) and Table 3.2.5(2). Also, the calculation method and requirements corresponding to each failure mode are given in the table.

Buckling Failure Modes and Theoretical Method Requirements for Stiffened Plate Panels Table 3.2.5 (1)

Analytical model	Failure mode		Calculation method and requirements
	Description	Buckling utilization factor	
（SP-A）	Overall buckling of	$\eta_{overall}$	η_{SP-A} - Maximum stiffened panel utilization

Method A for stiffened plate panel (SP-A)	stiffened plate panel		factor taken as the maximum of: ① The overall stiffened panel capacity as defined in 3.3.1; ② The plate buckling capacity calculated according to Method A as defined in 3.3.2(1); ③ The stiffener buckling strength as defined in 3.3.3 considering separately the properties (thickness, dimensions), the lateral pressures defined in 2.4.1 and the reference stresses of each elementary plate panel at both sides of the stiffener. Note: The stiffener buckling capacity check can only be fulfilled when the overall stiffened panel capacity, as defined in 3.3.1, is satisfied.
	Plate panel buckling according to Method A	η_{plate}	
	For stiffener buckling, the section properties (thickness and dimension) and reference stress of stiffeners on both sides of each elementary plate panel (EPP) shall be considered respectively in the calculation.	$\eta_{stiffener}$	
（SP-B）Method B for stiffened plate panel (SP-B)	Overall buckling of stiffened plate panel	$\eta_{overall}$	η_{SP-B} - Maximum stiffened panel utilization factor taken as the maximum of: ① The overall stiffened panel capacity as defined in 3.3.1; ② The plate buckling capacity calculated according to Method A as defined in 3.3.2(1); ③ The stiffener buckling strength as defined in 3.3.3 considering separately the properties (thickness, dimensions) and the reference stresses of each elementary plate panel at both sides of the stiffener. Note: The stiffener buckling capacity check can only be fulfilled when the overall stiffened panel capacity, as defined in 3.3.1, is satisfied.
	Plate panel buckling according to Method B	η_{plate}	
	For stiffener buckling, the section properties (thickness and dimensions) and reference stress of stiffeners on both sides of each elementary plate panel (EPP) shall be considered respectively in the calculation.	$\eta_{stiffener}$	
（UP-A）Method A for unstiffened plate panel (UP-A)	Plate panel buckling according to Method A	η_{plate}	η_{UP-A} - The maximum plate buckling utilization factor calculated according to Method A defined in 3.3.2(1).
（UP-B）Method B for unstiffened plate panel (UP-B)	Plate panel buckling according to Method B	η_{plate}	η_{UP-B} - The maximum plate buckling utilization factor calculated according to Method B defined in 3.3.2(1).
Web in ways of openings	Plate panel buckling of web in way of openings	$\eta_{opening}$	$\eta_{opening}$ - Maximum web utilization factor in way of openings, as defined in 3.4.
Curved plate panel	Buckling of curved plate panel	$\eta_{curve\text{-}plate}$	$\eta_{curve\text{-}plate}$ - Maximum utilization factor for buckling of curved plate panels defined in 3.3.2(6).

Buckling Failure Modes and Theoretical Method Requirements for Pillars, Struts, Cross Ties and Corrugated Bulkheads Table 3.2.5 (2)

Analytical	Failure mode	Calculation method and requirements

<table>
<tr><td>model</td><td>Description</td><td>Buckling utilization factor</td><td></td></tr>
<tr><td>Pillars, struts and cross ties</td><td>Elastic column buckling
Elastic torsional buckling
Elastic torsion + column buckling</td><td>η_{pillar}</td><td>η_{pillar} - Maximum buckling utilization factor
η_{pillar} for pillars, struts and cross ties as defined in 3.5.</td></tr>
<tr><td rowspan="3">Corrugated bulkheads</td><td>Overall column buckling</td><td>$\eta_{overall}$</td><td rowspan="3">$\eta_{overall}$ - Calculated according to the overall column buckling calculation method of corrugation unit defined in 3.6.1.
η_{Corr_p}, η_{Corr_w} - Calculated according to the method for calculating local buckling of corrugated bulkhead face plates and webs as defined in 3.6.2.</td></tr>
<tr><td>Local buckling of corrugation face plate</td><td>η_{Corr_p}</td></tr>
<tr><td>Local buckling of corrugation web</td><td>η_{Corr_w}</td></tr>
</table>

3.3 Plate panels and stiffeners

3.3.1 **Overall stiffened panel capacity**

The analysis of the overall elastic buckling capacity of a stiffened plate panel is to be based on the following interaction formula:

$$\frac{P_Z}{c_f} = 1$$

Buckling utilization factor: $\eta_{overall} = \frac{1}{\gamma_c}$

Where: c_f and P_Z - as shown in 3.3.3(4) of this Chapter;

γ_c - By solving the P_Z formula in 3.3.3 (4) of this Chapter, the value γ that meets the interaction formula is obtained.

3.3.2 **Plate buckling capacity**

(1) Plate limit state

The plate limit state is based on the following interaction formulae:

$$\left(\frac{\gamma_{c1}\sigma_x S}{\sigma_{cx}}\right)^{e_0} - B\left(\frac{\gamma_{c1}\sigma_x S}{\sigma_{cx}}\right)^{e_0/2}\left(\frac{\gamma_{c1}\sigma_y S}{\sigma_{cy}}\right)^{e_0/2} + \left(\frac{\gamma_{c1}\sigma_y S}{\sigma_{cy}}\right)^{e_0} + \left(\frac{\gamma_{c1}|\tau| S}{\tau_c}\right)^{e_0} = 1$$

$$\left(\frac{\gamma_{c2}\sigma_x S}{\sigma_{cx}}\right)^{2/\beta_p^{0.25}} + \left(\frac{\gamma_{c2}|\tau| S}{\tau_c}\right)^{2/\beta_p^{0.25}} = 1 \quad \text{for } \sigma_x \geq 0$$

$$\left(\frac{\gamma_{c3}\sigma_y S}{\sigma_{cy}}\right)^{2/\beta_p^{0.25}} + \left(\frac{\gamma_{c3}|\tau| S}{\tau_c}\right)^{2/\beta_p^{0.25}} = 1 \quad \text{for } \sigma_y \geq 0$$

$$\frac{\gamma_{c4}|\tau| S}{\tau_c} = 1$$

Then:

$$\gamma_c = \min(\gamma_{c1}, \gamma_{c2}, \gamma_{c3}, \gamma_{c4})$$

Buckling utilization factor: $\eta_{plate} = \dfrac{1}{\gamma_c}$

Where: σ_x, σ_y - Applied normal stress to the boundary of the plate panel, in N/mm^2, to be taken as defined in 3.3.2(7) of this Chapter;

τ - Applied shear stress to the boundary of the plate panel, in N/mm^2;

σ_{cx} - Ultimate buckling stress, in N/mm^2, in direction parallel to the longer edge of the buckling plate panel as defined in 3.3.2(3) of this Chapter;

σ_{cy} - Ultimate buckling stress, in N/mm^2, in direction parallel to the shorter edge of the buckling plate panel as defined in 3.3.2(3) of this Chapter;

$\gamma_{c1}, \gamma_{c2}, \gamma_{c3}, \gamma_{c4}$ - Stress multiplier factors at failure for each of the above different limit states. γ_{c2} and γ_{c3} are only to be considered when $\sigma_x \geq 0$ and $\sigma_y \geq 0$ respectively;

B - Coefficient given in Table 3.3.2(1);

e_0 - Coefficient given in Table 3.3.2(1);

β_p - Plate slenderness parameter taken as

$$\beta_p = \max\left(\frac{b}{t_p}\sqrt{\frac{R_{eH_P}}{E}}, 1.0\right)$$

Definition of Coefficients B and e_0 Table 3.3.2(1)

Applied stress	B	e_0
$\sigma_x \geq 0$ and $\sigma_y \geq 0$	$0.7 - 0.3\beta_p/\alpha^2$	$2/\beta_p^{0.25}$
$\sigma_x < 0$ or $\sigma_y < 0$	1.0	2.0

(2) Reference degree of slenderness

The reference degree of slenderness is to be taken as:

$$\lambda = \sqrt{\frac{R_{eH_P}}{K\sigma_E}}$$

Where: K - Buckling factor, as defined in Table 1.2.1 and Table 1.3.1 of Appendix 2 to this Chapter.

(3) The ultimate buckling stress of plate panel under axial stress, in N/mm^2, is to be taken as:

$$\sigma_{cx} = C_x R_{eH_P}$$
$$\sigma_{cy} = C_y R_{eH_P}$$

The ultimate buckling stress of plate panel subject to shear, in N/mm^2, is to be taken as:

$$\tau_c = C_\tau \frac{R_{eH_P}}{\sqrt{3}}$$

Where: C_x, C_y, C_τ - Reduction factor, as defined in Table 1.2.1 of Appendix 2 to this Chapter.

① For the 1st Equation of 3.1.3.2(1), when $\sigma_x < 0$ or $\sigma_y < 0$, the reduction factors are to be taken as: $C_x = C_y = C_\tau = 1$

② For other cases:

(a) For SP-A and UP-A, C_y is calculated according to Table 1.2.1 of Appendix 2 by using c_1:

$$c_1 = \left(1 - \frac{1}{\alpha}\right) \geq 0$$

(b) For SP-B and UP-B, C_y is calculated according to Table 1.2.1 of Appendix 2 by using c_1:

$$c_1 = 1$$

The boundary conditions for plates are to be considered as simply supported, see cases ①, ② and ⑮ in Table 1.2.1 of Appendix 2 to this Chapter. If the boundary conditions differ significantly from simple support, a more appropriate boundary condition can be applied according to the different cases of Table 1.2.1 of Appendix 2 to this Chapter subject to the agreement of CCS.

(4) Correction factor F_{long}

The correction factor F_{long} depending on the edge stiffener types on the longer side of the buckling plate panel is defined in Table 3.3.2(2). An average value of F_{long} is to be used for plate panels having different edge stiffeners. For stiffener types other than those mentioned in Table 3.3.2(2), the value of *c* is to be agreed by CCS. In such a case, value of *c* higher than those mentioned in Table 3.3.2(2) can be used, provided it is verified by buckling strength check of panel using non-linear FE analysis and deemed appropriate by CCS.

Correction factor F_{long} Table 3.3.2(2)

<table>
<tr><td colspan="3">Structural element types</td><td>F_{long}</td><td>c</td></tr>
<tr><td colspan="3">Unstiffened panel</td><td>1.0</td><td>--</td></tr>
<tr><td rowspan="6">Stiffened panel</td><td colspan="2">Stiffener not fixed at both ends</td><td>1.0</td><td>--</td></tr>
<tr><td rowspan="5">Stiffener fixed at both ends</td><td>Flat bar (1)</td><td rowspan="4">If $\frac{t_w}{t_p}>1$, then $F_{long}=c+1$;
If $\frac{t_w}{t_p}\le 1$, then $F_{long}=c\left(\frac{t_w}{t_p}\right)^3+1$</td><td>0.1</td></tr>
<tr><td>Bulb profile</td><td>0.3</td></tr>
<tr><td>Angle, L2 and L3 profile</td><td>0.4</td></tr>
<tr><td>T profile</td><td>0.3</td></tr>
<tr><td>Girder of high rigidity (e.g. bottom transverse)</td><td>1.4</td><td>N/A</td></tr>
<tr><td colspan="5">Note: t_w is the web thickness, in mm, without the correction defined in 3.3.3(2) of this Chapter.</td></tr>
</table>

(5) Correction factor F_{tran}

The correction factor F_{tran} is to be taken as 1.0.

(6) Curved plate panel

This requirement for curved plate panel limit state is applicable when $R/t \le 2500$. Otherwise, the requirement for plate limit state given in 3.3.2(1) of this Chapter is applicable.

The curved plate panel limit state is based on the following interaction formula:

$$\left(\frac{\gamma_c\sigma_{ax}S}{C_{ax}R_{eH_P}}\right)^{1.25}-0.5\cdot\left(\frac{\gamma_c\sigma_{ax}S}{C_{ax}R_{eH_P}}\right)\left(\frac{\gamma_c\sigma_{tg}S}{C_{tg}R_{eH_P}}\right)+\left(\frac{\gamma_c\sigma_{tg}S}{C_{tg}R_{eH_P}}\right)^{1.25}+\left(\frac{\gamma_c\tau\sqrt{3}S}{C_{\tau}R_{eH_P}}\right)^2=1.0$$

Buckling utilization factor: $\eta_{curve-plate}=\frac{1}{\gamma_c}$

Where: σ_{ax} - Applied axial stress to the curved plate panel, in N/mm^2. In case of tensile axial stresses, $\sigma_{ax}=0$;

σ_{tg} - Applied tangential stress to the curved plate panel, in N/mm^2. In case of tensile tangential stresses, $\sigma_{tg} = 0$;

C_{ax}, C_{tg}, C_{τ} - Buckling reduction factor of the curved plate panel, as defined in Table 1.3.1 of Appendix 2.

The stress multiplier factor, γ_c , of the curved plate panel need not be taken less than the stress multiplier factor, γ_c , for the expanded plane panel according to 3.3.2(1) of this Chapter.

(7) Applied axial stress and shear stress to the plate panel

The bi-axial stress, σ_x and σ_y , in N/mm^2, to be applied for the plate panel capacity calculation as given in 3.3.2(1) of the Chapter are to be taken as:

① For FE analysis, the reference stresses as defined in 2.6 of Chapter 2;

② For grillage beam analysis, the stresses are to be taken as:

$$\sigma_x = \frac{\sigma_{xb} + \nu\sigma_{yb}}{1-\nu^2}$$

$$\sigma_y = \frac{\sigma_{yb} + \nu\sigma_{xb}}{1-\nu^2}$$

Where: σ_{xb}, σ_{yb} - Stress, in N/mm^2, from grillage beam analysis respectively along *x* or *y* axis of the attached plate panel;

τ- Shear stress, in N/mm2, to be applied for the plate panel capacity calculation as given in 3.3.2(1) is to be taken:

(a) For FE analysis, the reference shear stress as defined in 3.3.3(4) of this Chapter;

(b) For grillage beam analysis, the shear stress $\tau=0$.

3.3.3 Stiffener buckling capacity

(1) Buckling modes

The following two buckling modes are to checked:

① Stiffener induced failure (SI);

② Associated attached plating induced failure (PI).

(2) Web thickness of flat bar

For accounting the decrease of the stiffness due to local lateral deformation, the effective web thickness t_{w_red} of flat bar stiffener, is to be used in 3.3.3(4) of this Chapter for the calculation of the sectional area, A_s , the modulus, Z , and the moment of inertia, I , of the stiffener and is taken as:

$$t_{w_red} = t_w \left(1 - \frac{2\pi^2}{3}\left(\frac{h_w}{s}\right)^2 \left(1 - \frac{b_{eff1}}{s}\right)\right) \text{mm}$$

(3) Idealization of bulb profile

Bulb profiles are to be considered as equivalent angle profiles. Equivalent angle profile size, in mm, is to be taken as:

$$h_w = h'_w - \frac{h'_w}{9.2} + 2$$

$$b_f = \alpha\left(t'_w + \frac{h'_w}{6.7} - 2\right)$$

$$t_f = \frac{h'_w}{9.2} - 2$$

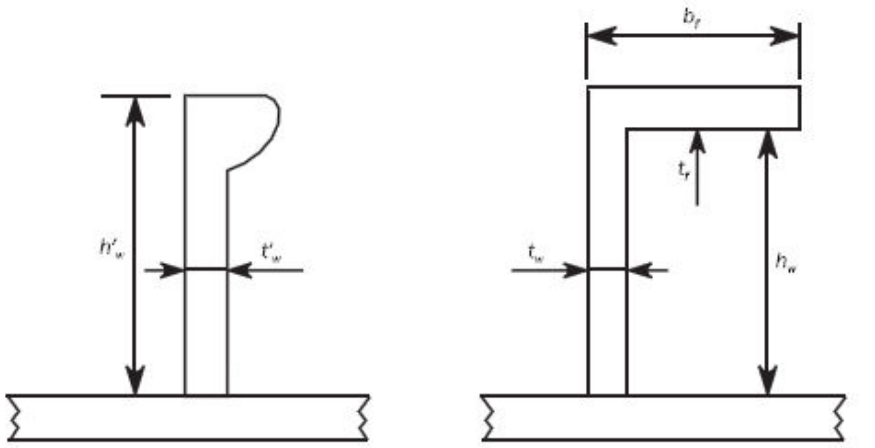
$$t_w = t'_w$$

Where: h'_w, t'_w - Height and thickness of bulb profile, in mm, as shown in Figure 3.3.3;

α - *Coefficient, to be taken as:*

$$\alpha = 1.1 + \frac{(120 - h'_w)^2}{3000} \quad \text{for } h'_w \le 120$$

$$\alpha = 1.0 \quad \text{for } h'_w \le 120$$



Bulb profile, Equivalent angle profile

Figure 3.3.3 Equivalent Angle Profile for Bulb Profile

(4) Ultimate buckling capacity

When $\sigma_a + \sigma_b + \sigma_w > 0$, the ultimate buckling capacity for stiffeners is to be checked according to the following interaction formula:

$$\frac{\gamma_c \sigma_a + \sigma_b + \sigma_w}{R_{eH}} S = 1$$

Buckling utilization factor: $\eta_{stiffener} = \frac{1}{\gamma_c}$

Where: σ_a - Effective axial stress, in N/mm^2, at mid span of the stiffener, acting on the stiffener with its attached plating, to be taken as:

$$\sigma_a = \sigma_x \frac{st_p + A_s}{b_{eff1}t_p + A_s}$$

σ_x - Nominal axial stress, in N/mm^2, acting on the stiffener with its attached plating, to be taken as:

(a) For FE analysis, σ_x is the FE corrected stress as defined in 3.3.2(7) of this Chapter in the attached plating in the direction of the stiffener axis;

(b) For grillage beam analysis, σ_x is the stress acting along the x-axis of the attached buckling plate panel;

R_{eH} - Yield stress of material, in N/mm^2, to be taken as:

(a) $R_{eH} = R_{eH_S}$ for stiffener induced failure (SI);

(b) $R_{eH} = R_{eH_P}$ for attached plating induced failure (PI);

σ_b - Bending stress in the stiffener, in N/mm^2, to be taken as:

$$\sigma_b = \frac{M_o + M_1}{1000Z}$$

Z- Section modulus of stiffener, in cm^3, including effective width of attached plating according to 3.3.3(5) of this Chapter, to be taken as:

(a) The section modulus calculated at the top of stiffener flange for stiffener induced failure (SI);

(b) The section modulus calculated at the attached plating for attached plating induced failure (PI);

C_{PI} - Attached plating induced failure pressure coefficient, to be taken as:

(a) $C_{PI} = 1$ if the lateral pressure is applied on the side opposite to the stiffener;

(b) $C_{PI} = -1$ if the lateral pressure is applied on the same side as the stiffener;

C_{SI} - Stiffener induced failure pressure coefficient, to be taken as:

(a) $C_{SI} = -1$ if the lateral pressure is applied on the side opposite to the stiffener;

(b) $C_{SI} = 1$ if the lateral pressure is applied on the same side as the stiffener;

M_1 - Bending moment, in $\mathrm{N \cdot mm}$, due to the lateral load *P,* to be taken as:

(a) For continuous stiffener:

$$M_1 = C_i \frac{|P| s l_s^2}{24 \times 10^3}$$

(b) For sniped stiffener at both ends:

$$M_1 = C_i \frac{|P| s l_s^2}{8 \times 10^3}$$

(c) For stiffener sniped at one end and continuous at the other end:

$$M_1 = C_i \frac{|P| s l_s^2}{14.2 \times 10^3}$$

P - Lateral load, in kN/m^2, is the average pressure as defined in 2.8 of Chapter 2 in the attached plating;

C_i - Pressure coefficient, to be taken as:

(a) $C_i = C_{SI}$ for stiffener induced failure (SI);

(b) $C_i = C_{PI}$ for attached plating induced failure (PI);

M_0 - Bending moment, in $\mathrm{N \cdot mm}$, due to the lateral deformation w of stiffener, to be taken as:

$$M_0 = F_E \left(\frac{P_z w}{c_f - P_z} \right), \text{ and } c_f - P_z > 0$$

F_E - Ideal elastic buckling force of the stiffener, in N, to be taken as:

$$F = \left(\frac{\pi}{l_s} \right) EI \times 10^4$$

I - Moment of inertia, in cm^4, of the stiffener including effective width of attached plating according to 3.3.3(5) of the Chapter. It is to comply with the following requirement:

$$I \geq \frac{s t_p^3}{12 \times 10^4}$$

t_p - Slab thickness, in mm, defined as the calculated elementary plate panel thickness at one side of stiffener;

P_z - Nominal lateral load, in N/mm^2, acting on the stiffener due to stresses, σ_x, σ_y and τ, in the attached plating in way of the stiffener mid span, to be taken as:

$$P_z = \frac{t_p}{s} \left(\sigma_{xl} \left(\frac{\pi s}{l_s} \right)^2 + 2c\gamma\sigma_y + \sqrt{2}\tau_1 \right)$$

Where: $\sigma_{xl} = \gamma\sigma_x \left(1 + \frac{A_s}{s t_p} \right) \geq 0$

$$\tau_1 = \gamma|\tau| - t_p \sqrt{R_{eH_P} E \left(\frac{m_1}{a^2} + \frac{m_2}{b^2} \right)} \geq 0$$

σ_y - Stress applied on the edge along y axis of the buckling plate panel, in N/mm^2, but not less than 0, and;

(a) For FE analysis, σ_y is the FE corrected stress as defined in 3.3.3(6) of this Chapter in the attached plating in the direction perpendicular to the stiffener;

(b) For grillage beam analysis, σ_y is the stress acting along the y-axis of the attached buckling plate panel;

τ - Applied shear stress, in N/mm^2, and:

(a) For FE analysis, τ is the reference shear stress as defined in 2.7 of Chapter 2 in the attached plating;

(b) For grillage beam analysis, $\tau = 0$ in the attached buckling plate panel;

m_1 , m_2 - Coefficients to be taken as:

$$\begin{cases} m_1 = 1.47 & m_2 = 0.49 \ \textbf{when} \ \alpha \geq 2 \\ m_1 = 1.96 & m_2 = 0.37 \ \textbf{when} \ \alpha < 2 \end{cases}$$

c - Factor taking into account the stresses in the attached plating acting perpendicular to the stiffener's axis, to be taken as:

$$c = \begin{cases} 0.5(1+\psi) \ \textbf{when} \ \ 0 \leq \psi \leq 1 \\ \dfrac{1}{2(1-\psi)} \ \ \textbf{when} \ \ \psi < 0 \end{cases}$$

ψ - Edge stress ratio for case ② according to Table 1.2.1 of Appendix 2;

w - Deformation of stiffener, in mm, to be taken as:

$$w = w_0 + w_1$$

Where: w_0 - Assumed imperfection, in mm, to be taken as:

(a) In general:

$w_0 = l_s / 1000$;

$w_0 = l_s / 1000$;

(b) For stiffeners sniped at one or both ends considering stiffener induced failure (SI):

$$w_0 = -w_{na}$$

(c) For stiffeners sniped at one or both ends considering attached plating induced failure (PI):

$$w_0 = w_{na}$$

w_{na} - Distance from the mid-point of attached plating to the neutral axis of the stiffener calculated with the effective width of the attached plating according to 3.3.3(5) of this Chapter, in mm;

w_1 - Deformation of stiffener, in mm, at mid-point of stiffener span due to lateral load P. In case of uniformly distributed load, w_1 is to be taken as:

(a) $w_1 = C_i \dfrac{|P| s l_s^4}{384 \times 10^7 EI}$ in general;

(b) $w_1 = C_i \dfrac{5|P| s l_s^4}{384 \times 10^7 EI}$ for sniped stiffener at both ends;

c_f - Elastic support provided by the stiffener, in N/mm^2, to be taken as:

$$c_f = F_E\left(\frac{\pi}{l_s}\right)^2 (1+c_p)$$

$$c_p = \frac{1}{1+\frac{0.91}{c_{xa}}\left(\frac{12I}{st_p^3}\times 10^4 - 1\right)}$$

c_{xa} - Coefficients to be taken as:

(a) For $l_s \geq 2s$, $c_{xa} = \left[\frac{l_s}{2s} + \frac{2s}{l_s}\right]^2$;

(b) For $l_s < 2s$, $c_{xa} = \left[1+\left(\frac{l_s}{2s}\right)^2\right]^2$

σ_w - Stress due to torsional deformation, in N/mm^2, to be taken as:

(a) For stiffener induced failure (SI):

$$\sigma_w = Ey_w\left(\frac{t_f}{2}+h_w\right)\Phi_0\left(\frac{\pi}{l_s}\right)^2\left(\frac{1}{1-\frac{0.4R_{eH_S}}{\sigma_{ET}}}-1\right);$$

$$\sigma_w = Ey_w\left(\frac{t_f}{2}+h_w\right)\Phi_0\left(\frac{\pi}{l_s}\right)^2\left(\frac{1}{1-\frac{0.4R_{eH_S}}{\sigma_{ET}}}-1\right);$$

(b) For attached plating induced failure (PI):

$$\sigma_w = 0;$$

y_w - Distance, in mm, from centroid of stiffener cross section to the free edge of stiffener flange, to be taken as:

(a) $y_w = \frac{t_w}{2}$ for flat bar;

(b) $y_w = b_f - \frac{h_w t_w^2 + t_f b_f^2}{2A_s}$ for angle and bulb profiles;

(c) $y_w = b_{f-out} + 0.5t_w - \frac{h_w t_w^2 + t_f(b_f^2 - 2b_f d_f)}{2A_s}$ for L2 profile;

(d) $y_w = b_{f-out} + 0.5t_w - \frac{(h_w - t_f)t_w^2 + t_f(b_f + t_w)^2}{2A_s}$ for L3 profile;

(e) $y_w = \frac{b_f}{2}$ for T profile;

Φ_0 - Coefficient to be taken as:

$$\Phi_0 = \frac{l_s}{h_w} \times 10^{-3}$$

σ_{ET} - Reference stress for torsional buckling, in N/mm^2, to be taken as:

$$\sigma_{ET} = \frac{E}{I_p}\left(\frac{\varepsilon \pi^2 I_\omega \times 10^2}{l_s^2} + 0.385 I_T\right)$$

Where: I_p - Polar moment of inertia of the stiffener, in cm^4, about point C as shown in Figure 3.3.1, as defined in Table 3.3.4;

I_T - St. Venant's moment of inertia of the stiffener, in cm^4, as defined in Table 3.3.3;

I_ω - Sectional moment of inertia of the stiffener, in cm^6, about point C as shown in Figure 3.3.1, as defined in Table 3.3.3;

ε - Degree of fixation, to be taken as:

$$\varepsilon = 1 + \frac{\left(\frac{l_s}{\pi}\right)^2 \times 10^{-3}}{\sqrt{I_\omega\left(\frac{0.75s}{t_p^3} + \frac{e_f - 0.5t_f}{t_w^3}\right)}}$$

A_w - Web area, in mm^2;

A_f - Flange area, in mm^2;

Inertia Moment of Typical Cross Sections Table 3.3.3

Cross Sectional Properties	Flat bar (1)	Bulb profile, angle profile, L2, L3 and T profiles
I_P	$\frac{h_w^3 t_w}{3 \cdot 10^4}$	$\left(\frac{A_w (e_f - 0.5t_f)^2}{3} + A_f e_f^2 \right) 10^{-4}$
I_T	$\frac{h_w t_w^3}{3 \cdot 10^4} \left(1 - 0.63 \frac{t_w}{h_w} \right)$	$\frac{(e_f - 0.5t_f) t_w^3}{3 \cdot 10^4} \left(1 - 0.63 \frac{t_w}{e_f - 0.5t_f} \right) + \frac{b_f t_f^3}{3 \cdot 10^4} \left(1 - 0.63 \frac{t_f}{b_f} \right)$
I_ω	$\frac{h_w^3 t_w^3}{36 \cdot 10^6}$	For bulb profile, angle profile, L2 and L3 profiles: $\frac{A_f e_f^2 b_f^2}{12 \cdot 10^6} \left(\frac{A_f + 2.6 A_w}{A_f + A_w} \right)$ For T profile: $\frac{b_f^3 t_f e_f^2}{12 \cdot 10^6}$
Note(1): t_w is the web thickness, in mm. t_{w_red} as defined in 3.3.3(2) is not to be used in this table.		

(5) Effective width of attached plating

The effective width of attached plating of stiffeners, beff, b_{eff}, in mm, is to be taken as:

$$b_{eff} = \begin{cases} \min(C_x b, \chi_s s) & \text{When } \sigma_x > 0 \\ \chi_s s & \text{When } \sigma_x \le 0 \end{cases}$$

Where: χ_s - Effective width coefficient to be taken as:

$$\chi_s = \begin{cases} \min \left[\frac{1.12}{1 + \frac{1.75}{\left(\frac{l_{eff}}{s} \right)^{1.6}}}; 1.0 \right] & \text{When } \frac{l_{eff}}{s} \ge 1 \\ 0.407 \frac{l_{eff}}{s} & \text{When } \frac{l_{eff}}{s} < 1 \end{cases}$$

l_{eff} - Effective length of the stiffener, in mm, to be taken as:

(a) $l_{eff} = \frac{l_s}{\sqrt{3}}$ for stiffener fixed at both ends;

(b) $l_{eff} = 0.75 l_s$ stiffener simply supported at one end and fixed at the other;

(c) $l_{eff} = l_s$ for stiffener simply supported at both ends;

(6) FE corrected stresses for stiffener capacity

When the reference stresses σ_x and σ_y obtained by FE analysis according to 2.7 of Chapter 2 are both compressive, they are to be corrected according to the following formulae:

If $\sigma_x < \nu\sigma_y$,

$$\sigma_{xcor} = 0$$

$$\sigma_{ycor} = \sigma_y$$

If $\sigma_y < \nu\sigma_x$,

$$\sigma_{xcor} = \sigma_x$$

$$\sigma_{ycor} = 0$$

In other cases:

$$\sigma_{xcor} = \sigma_x - \nu\sigma_y$$

$$\sigma_{ycor} = \sigma_y - \nu\sigma_x$$

3.4 Web of primary supporting members

3.4.1 Web in ways of openings

The web of primary supporting members with openings is to be assessed for buckling based on the combined axial compressive and shear stresses.

The web adjacent to the opening on both sides is to be considered as individual unstiffened plate panels as shown in Table 3.4.2.

The interaction formulae of 3.3.2(1) are to be used with:

$$\sigma_x = \sigma_{av}$$
$$\sigma_y = 0$$
$$\tau = \tau_{av}$$

Buckling utilization factor: $\eta_{opening} = \dfrac{1}{\gamma_c}$

Where: σ_{av} - Weighted average compressive stress, in N/mm^2, in the area of web being considered, i.e. ①, ② or ③ as shown in Table 1.2.1 of Appendix 2;

γ_c - See 3.3.2(1) of this Chapter.

For the application of the Table 3.4.2, the weighted average shear stress is to be taken as:

① Opening modelled in primary supporting members:

τ_{av} - Weighted average shear stress, in N/mm^2, in the area of web being considered , i.e. P1, P2, or P3 as shown in Table 3.4.2.

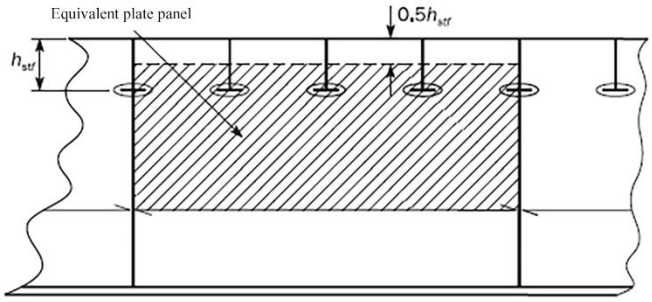
② Opening not modelled in primary supporting members:

τ_{av} - Weighted average shear stress, in N/mm^2, given in Table 3.4.2.

3.4.2 Reduction factors of web in way of openings

The reduction factors, C_x or C_y in combination with, C_τ of the plate panel(s) of the web adjacent to the opening is to be taken as shown in Table 3.4.2.

3.4.3 The equivalent plate panel of web of primary supporting members crossed by perpendicular stiffeners is to be idealised as shown in Figure 3.4.3.

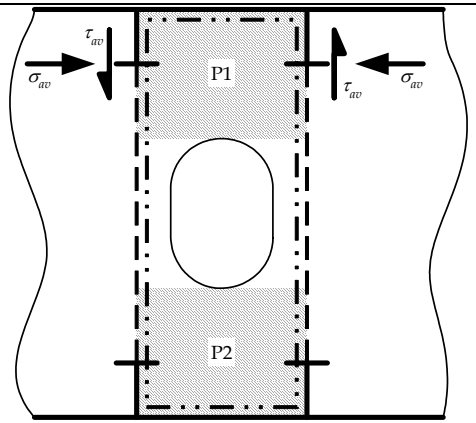


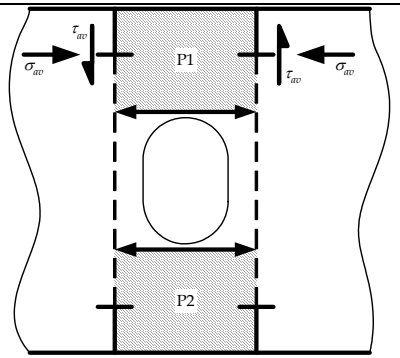
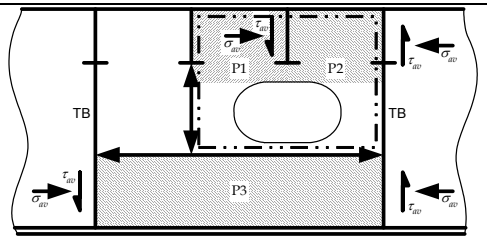
Note: The correction for plate panel width also applies to other opening structures, provided that the web and ring plates are at least connected to one side passing through the stiffeners.

Figure 3.4.3 Idealisation of Web of Primary Supporting Members

3.4.4 For the web buckling assessment of primary supporting members other than those specified in 3.4.1, UP-B is generally to be used except where indicated in Table 2.3.1 and Figure 2.3.1(1) to (6). A more appropriate method of assessing plate panel buckling may also be adopted, subject to agreement with CCS, depending on the actual arrangement of the structure.

Buckling Reduction Factor Table 3.4.2

Structure Configuration		C_x, C_y	C_τ	
			Opening modelled in supporting member	Opening not modelled in supporting member
(a) Without edge reinforcements		Separate reduction factors are to be applied to areas P1 and P2 using case ③ or case ⑥ in Table 1.2.1 of Appendix 2, with edge stress ratio: Ψ=1.0.	Separate reduction factors are to be applied to areas P1 and P2 using case ⑱ or case ⑲ in Table 1.2.1 of Appendix 2.	$\tau_{av} = \tau_{av}(web)$ When case ⑰ in Table 1.2.1 of Appendix 2 is applicable, a common reduction factor is to be applied to areas P1 and P2 using case ⑰ with $\tau_{av} = \tau_{av}(web)$; $\tau_{av} = \tau_{av}(web)h/(h-h_0)$ When case ⑰ in Table 1.2.1 of Appendix 2 is not applicable, separate reduction factors are to be applied to areas P1 and P2 using case ⑱ or case ⑲ with $\tau_{av} = \tau_{av}(web)h/(h-h_0)$

<table>
<tr>
<td>(b) With edge reinforcements</td>
<td>
</td>
<td>Separate reduction factors are to be applied for areas P1 and P2 using the following:
case ① for Cx or case ② for Cy in Table 1.2.1 of Appendix 2 with stress ratio: Ψ=1.0.</td>
<td>Separate reduction factors are to be applied for areas P1 and P2 using case ⑮ in Table 1.2.1 of Appendix 2</td>
<td>$\tau_{av} = \tau_{av}(web)h/(h-h_0)$
Separate reduction factors are to be applied to areas P1 and P2 using case ⑮ in Table 1.2.1 of Appendix 2 with:
$\tau_{av} = \tau_{av}(web)h/(h-h_0)$</td>
</tr>
<tr>
<td>(c) Example of openings in web</td>
<td>
</td>
<td colspan="3">Plate panels P1 and P2 are to be evaluated in accordance with (a). Plate panel P3 is to be evaluated in accordance with (b).</td>
</tr>
<tr>
<td></td>
<td colspan="4">Note: 1. Plate panel to be considered for buckling in way of openings are shown shaded and numbered P1, P2, etc.</td>
</tr>
<tr>
<td></td>
<td colspan="4">Where: h - Height, in m, of the web of the primary supporting member in way of the opening;
h_0 - Height in m, of the opening measured in the depth of the web;
$\tau_{av}(web)$ - Weighted average shear stress, in N/mm^2, over the web of the primary supporting member in the table.</td>
</tr>
</table>

3.5 Pillars, struts and cross ties

3.5.1 Buckling utilization factor

The buckling utilization factor, η_{pillar}, for axially compressed struts, pillars and cross ties is to be taken as:

$$\eta_{pillar} = \frac{\sigma_{av}}{\sigma_{cr}}$$

Where: σ_{av} - Average axial compressive stress in the member, in N/mm^2;

σ_{cr} - Minimum critical buckling stress, in N/mm^2, to be taken as:

① $\sigma_{cr} = \sigma_E$ for $\sigma_E \le 0.5R_{eH_S}$

② $\sigma_{cr} = \left(1 - \frac{R_{eH_S}}{4\sigma_E}\right) R_{eH_S}$ for $\sigma_E > 0.5R_{eH_S}$

σ_E - Minimum elastic compressive buckling stress, in N/mm^2, obtained by each buckling mode respectively, corresponding to σ_{EC}, σ_{EI} and σ_{ETF} in 3.5.2~3.5.4 of this Chapter respectively;

R_{eH_S} - Yield stress of the material, in N/mm^2. For built up members, the lowest minimum yield stress is to be used.

3.5.2 Elastic column buckling stress

The elastic compressive column buckling stress, σ_{EC} in N/mm2 of members subject to axial compression is to be taken as:

$$\sigma_{EC} = \pi^2 E f_{end} \frac{I}{A l_{pill}^2} \times 10^{-4}$$

Where: I - Minimum inertia moment of cross section, in cm^4;

A - Cross sectional area of the member, in cm^2;

l_{pill} - Length of the member, in mm, to be taken as:

① For pillar, strut and corrugated bulkhead corrugation: unsupported length of the member;

② For cross tie:

a. In centre tank: distance between the flanges of longitudinal stiffeners on the oard and port longitudinal bulkheads to which the cross tie's horizontal stringer is attached;

b. In wing tank: distance between the flanges of longitudinal stiffeners on the

longitudinal bulkhead to which the cross tie's horizontal stringer is attached, and the inner hull plating;

f_{end} - End constraint factor, to be taken as:

① For pillar and strut:

$f_{end} = 1.0$ where both ends are simply supported;

$f_{end} = 2.0$ where one end is simply supported and the other end is fixed;

$f_{end} = 4.0$ where both ends are fixed;

② For cross tie:

$f_{end} = 2.0$

③ For corrugated bulkhead corrugation:

The appropriate value of end constraint factor is to be selected according to the constraints at both ends of the corrugation. For either end, if the width of the stool exceeds 2 times the depth of the corrugation, it is considered to be fixed, otherwise it is simply supported.

A pillar end may be considered fixed when brackets of adequate size are fitted. Such brackets are to be sup- ported by structural members with greater bending stiffness than the pillar.

3.5.3 Elastic torsional buckling stress

The elastic torsional buckling stress, σ_{ET}, in N/mm^2, with respect to axial compression of members is to be taken as:

$$\sigma_{EI} = \frac{GI_{sv}}{I_{pol}} + \frac{\pi^2 f_{end} E c_{warp}}{I_{pol} l_{pill}^2} \times 10^{-4}$$

Where: I_{SV} - St. Venant's moment of inertia, in cm^4, see Table 3.5.3 for examples of cross sections;

I_{pol} - Polar moment of inertia about the shear centre of cross section, in cm^4, to be taken as:

$$I_{pol} = I_y + I_z + A\left(y_0^2 + z_0^2\right)$$

c_{warp} - Warping constant, in cm^6, see Table 3.5.3 for examples of cross sections;

ℓ_{pill} - Length of the member, in m, as defined in 3.5.2 of this Chapter;

y_0 - y coordinate of shear centre relative to the cross sectional centroid, in cm, see Table 3.5.3 for examples of cross sections.

z_0 - z coordinate of shear centre relative to the cross sectional centroid, in cm, see Table 3.5.3 for examples of cross sections.

A - Cross sectional area, in cm^2, as defined in 3.5.2 of this Chapter;

I_y - Moment of inertia about y axis, in cm^4;

I_z - Moment of inertia about z axis, in cm^4.

3.5.4 Elastic torsional/column buckling stress

For cross sections where the centroid and the shear centre do not coincide, the interaction between the torsional and column buckling mode is to be examined. The elastic torsional/column buckling stress, σ_{ETF}, with respect to axial compression is to be taken as:

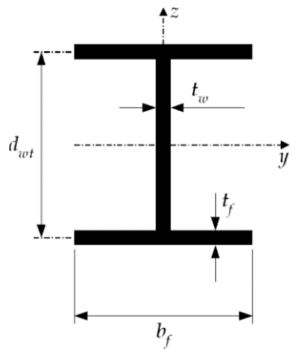
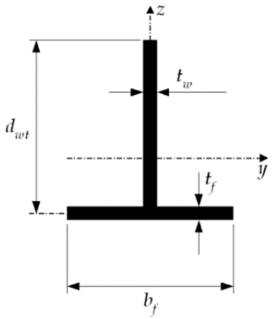
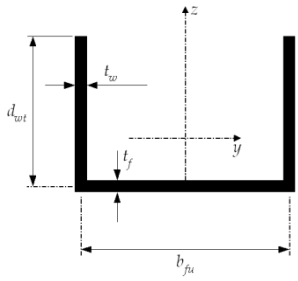
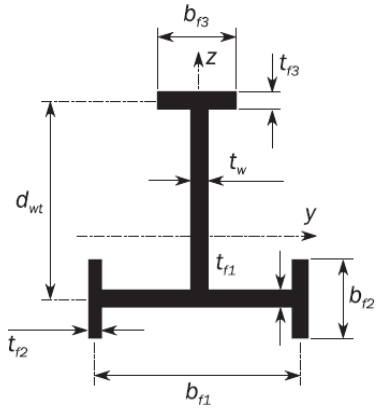
$$\sigma_{ETF} = \frac{1}{2\varsigma}\left[\left(\sigma_E + \sigma_{ET}\right) - \sqrt{\left(\sigma_E + \sigma_{ET}\right)^2 - 4\varsigma\sigma_E\sigma_{ET}}\right] \qquad \text{N/mm}^2$$

Where: ζ - Coefficient to be taken as:

$$\varsigma = 1 - \frac{\left(y_0^2 + z_0^2\right)A}{I_{pol}}$$

Remaining symbols—see 3.5.2 and 3.5.3 of this Chapter.

Typical Cross Sectional Properties Table 3.5.3

	$I_{sv} = \frac{1}{3}\left(2b_f t_f^3 + d_{wt} t_w^3\right)10^{-4}$ cm^4 $c_{warp} = \frac{d_{wt}^2 b_f^3 t_f}{24}10^{-6}$ cm^6
	$I_{sv} = \frac{1}{3}\left(b_f t_f^3 + d_{wt} t_w^3\right)10^{-4}$ cm^4 $y_0 = 0$ cm $z_0 = -\frac{0.5 d_{wt}^2 t_w}{d_{wt} t_w + b_f t_f}10^{-1}$ cm $c_{warp} = \frac{b_f^3 t_f^3 + 4 d_{wt}^3 t_w^3}{144}10^{-6}$ cm^6
	$I_{sv-net50} = \frac{1}{3}\left(b_{fu} t_f^3 + 2 d_{wt} t_w^3\right)10^{-4}$ cm^4 $y_0 = 0$ cm $z_0 = -\frac{d_{wt}^2 t_w 10^{-1}}{2 d_{wt} t_w + b_f t_f} - \frac{0.5 d_{wt}^2 t_w 10^{-1}}{d_{wt} t_w + b_{fu} t_f / 6}$ cm $c_{warp} = \frac{b_{fu}^2 d_{wt}^3 t_w \left(3 d_{wt} t_w + 2 b_{fu} t_f\right)}{12\left(6 d_{wt} t_w + b_{fu} t_f\right)}10^{-6}$ cm^6
	$I_{sv} = \frac{1}{3}\left(b_{f1} t_{f1}^3 + 2 b_{f2} t_{f2}^3 + b_{f3} t_{f3}^3 + d_{wt} t_w^3\right)10^{-4}$ cm^4 $y_0 = 0$ cm $z_o = z_s - \frac{(b_{f3} d_{wt} t_{f3} + 0.5 d_{wt}^2 t_w)10^{-1}}{d_{wt} t_w + b_{f1} t_{f1} + 2 b_{f2} t_{f2} + b_{f3} t_{f3}}$ cm $c_{warp} = \left(I_{f1} Z_s^2 + \frac{I_{f2} b_{f1}^2}{200} + I_{f3}\left(\frac{d_{wt}}{10} - Z_s\right)^2\right)$ cm^6

	$I_{f1} = (\frac{(b_{f1} - t_{f2})^3 t_{f1}}{12} + \frac{b_{f2} t_{f2} b_{f1}^2}{2}) 10^{-4}$ cm^4 $I_{f2} = \frac{b_{f2}^3 t_{f2}}{12} 10^{-4}$ cm^4 $I_{f3} = \frac{b_{f3}^3 t_{f3}}{12} 10^{-4}$ cm^4 $z_s = \frac{I_{f3} d_{wt}}{I_{f1} + I_{f3}} 10^{-1}$ cm
Note: (1) All dimensions are in mm; (2) Cross sectional properties are given for typical cross sections. Properties for other cross sections are to be determined by direct calculation.	

3.6 Corrugated bulkhead

3.6.1 Overall column buckling

The overall column buckling of a corrugated bulkhead is to be calculated for each corrugation unit consisting of 1/2 face plate + 1 web + 1/2 face plate, i.e., single corrugation as shown in grey in Figure 3.6.1

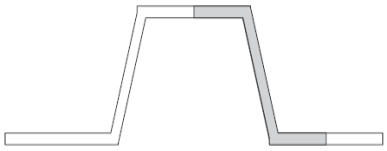


Figure 3.6.1 Schematic Diagram of Corrugation Unit/Single Corrugation

Column buckling capacity and buckling utilization factor $\eta_{overall}$ for corrugation unit are calculated using the formulae given in 3.5.2 and 3.5.1 of this Chapter, respectively. For each corrugation unit, the member length l_{pill} and end constraint factors f_{end} are defined in 3.5.2 of this Chapter.

3.6.2 Local buckling

The local buckling utilization factor η_{Corr} for face plate and web of corrugated bulkheads is to be based on a combination of in-plane and shear stresses. Two stress combinations are to be considered for the application of the calculation methods in 2.9.2 of Chapter 2:

① The maximum normal stress parallel to the corrugation, σ_x, combined with the stress perpendicular to the corrugation, σ_y, and with the shear stress, τ, at the location where the maximum normal stress parallel to the corrugation occurs;

② The maximum shear stress, τ, combined with the normal stress parallel to the corrugation, σ_x, and with the stress perpendicular to the corrugation, σ_y, at the location where the maximum shear stress occurs;

The buckling capacity formulae are those given in 3.3.2(1) of this Chapter, where the axial and shear ultimate buckling stresses are to be based on the buckling cases ①, ②, ⑤ from Table 1.2.1 of Appendix 2 and the relevant coefficients are defined as follows:

$$\alpha = 2$$

$$\psi_x = \psi_y = 1$$

3.7 CFM Calculation Software COMPASS-ABA-CFM

3.7.1 This software is a CFM calculation program for buckling and ultimate strength of stiffened plate structure developed by CCS according to the relevant requirements of buckling strength analysis of stiffened plate structure in this Chapter.

3.7.2 The software utilizes Excel as its interface and data platform, and incorporates secondary development using VBA. The main interface layout is illustrated in Figure 3.7.2.

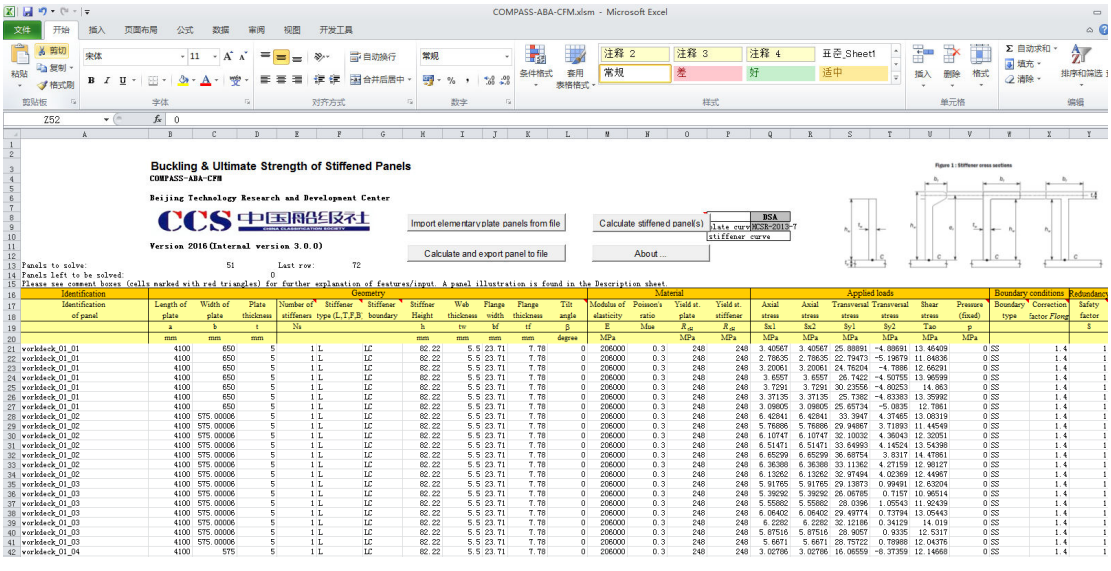


Figure 3.7.2 Main Interface of COMPASS-ABA-CFM Software

3.7.3 The software allows for input of plate panel information and load cases via the interface, or import of the above input data from external files.

3.7.4 The CCS ore carrier direct calculation software provides interface files that directly integrate with this software, enabling direct invocation for buckling assessments of applicable unstiffened/stiffened plate panels and load cases.

3.7.5 The user manual of the COMPASS-ABA-CFM software can be referred as detailed instructions.

Appendix 2 Buckling Factor and Reduction Factor for Plate Panels

1.1 Symbols

1.1.1 Explanations of symbols used in this Appendix refer to 3.1.1 of Chapter 3.

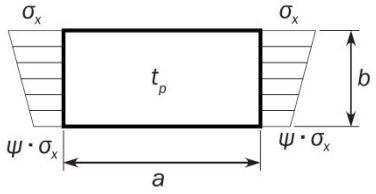
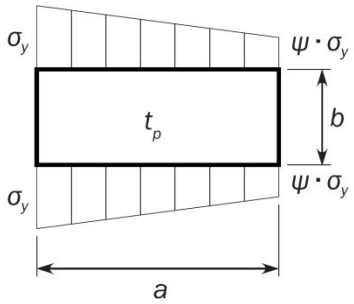
1.2 Plane plate panel

1.2.1 Buckling factor and reduction factor for plane plate panels are given in Table 1.2.1 of the Appendix.

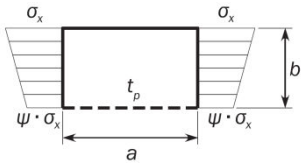
1.3 Curved plate panel

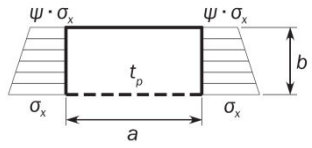
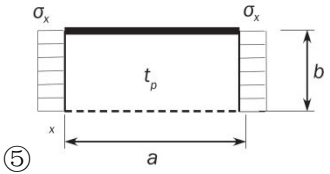
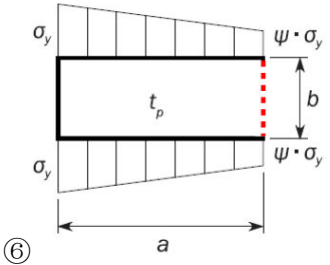
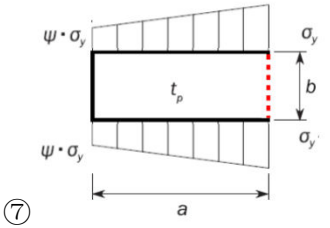
1.3.1 Buckling factor and reduction factor for curved plate panels satisfying the $R / t_p \leq 2500$ conditions are given in Table 1.3.1 of the Appendix.

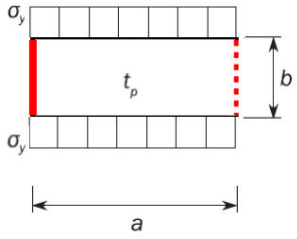
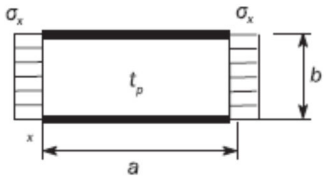
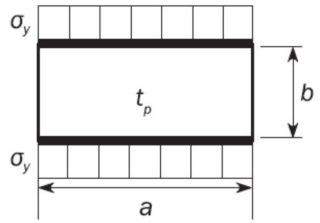
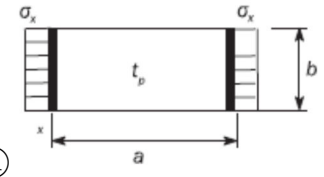
Buckling Factor and Reduction Factor for Plane Plate Panels Table 1.2.1

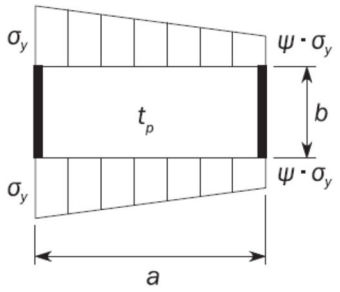
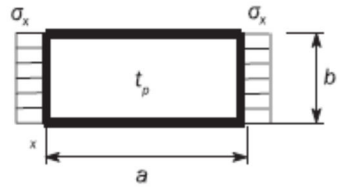
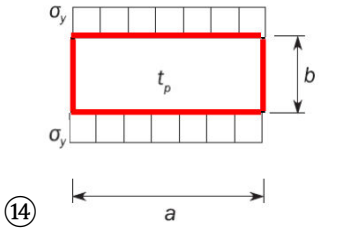
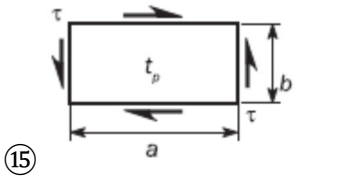
Case	Stress ratio ψ	Aspect ratio α	Buckling factor K	Reduction factor C
① 	$1 \ge \psi \ge 0$	$K_x = F_{long}\dfrac{8.4}{\psi+1.1}$		When $\sigma_x \le 0$, $C_x = 1$; When $\sigma_x > 0$: $C_x = 1$ for $\lambda \le \lambda_c$; $C_x = c\left(\dfrac{1}{\lambda}-\dfrac{0.22}{\lambda^2}\right)$ for $\lambda > \lambda_c$ Where: $c = (1.25-0.12\psi) \le 1.25$ $\lambda_c = \dfrac{c}{2}\left(1+\sqrt{1-\dfrac{0.88}{c}}\right)$
	$0 > \psi > -1$	$K_x = F_{long}\left[7.63-\psi(6.26-10\psi)\right]$		
	$\psi \le -1$	$K_x = F_{long}\left[5.975(1-\psi)^2\right]$		
② 	$1 \ge \psi \ge 0$	$K_y = F_{tran}\dfrac{2\left(1+\frac{1}{\alpha^2}\right)^2}{1+\psi+\frac{(1-\psi)}{100}\left(\frac{2.4}{\alpha^2}+6.9f_1\right)}$		When $\sigma_y \le 0$, $C_y = 1$; When $\sigma_y > 0$: $C_y = c\left(\dfrac{1}{\lambda}-\dfrac{R+F^2(H-R)}{\lambda^2}\right)$ Where: $c = (1.25-0.12\psi) \le 1.25$ $R = \lambda(1-\lambda/c)$ for $\lambda < \lambda_c$ $R = 0.22$ for $\lambda \ge \lambda_c$ $\lambda_c = 0.5c(1+\sqrt{1-0.88/c})$
		$\alpha \le 6$	$f_1 = (1-\psi)(\alpha-1)$	
		$\alpha > 6$	$f_1 = 0.6(1-\dfrac{6\psi}{\alpha})(\alpha+\dfrac{14}{\alpha})$ but not greater than $14.5-\dfrac{0.35}{\alpha^2}$	
	$1-\dfrac{4\alpha}{3} \le \psi < 0$	$K_y = \dfrac{200F_{tran}\left(1+\beta^2\right)^2}{(1-f_3)(100+2.4\beta^2+6.9f_1+23f_2)}$		

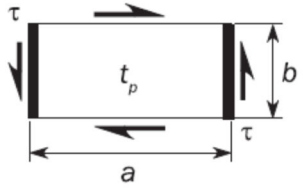
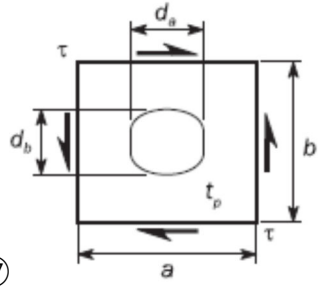
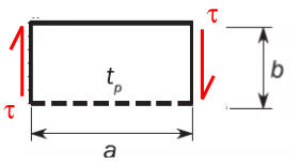
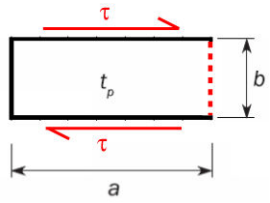
		$\alpha > 6(1-\psi)$	$f_1 = 0.6(\frac{1}{\beta}+14\beta)$ but not greater than $14.5-0.35\beta^2$ $f_2 = f_3 = 0$	$F = \left(1-\left(\frac{K}{0.91}-1\right)\Big/\lambda_p^2\right)c_1 \geq 0$ $\lambda_p^2 = \lambda^2 - 0.5$ **and** $1 \leq \lambda_p^2 \leq 3$ $c_1 = 1$ *as defined in 3.3.2(3)②.* $H = \lambda - \frac{2\lambda}{c(T+\sqrt{T^2-4})} \geq R$ $T = \lambda + \frac{14}{15\lambda} + \frac{1}{3}$
		$3(1-\psi) \leq \alpha \leq 6(1-\psi)$	$f_1 = \frac{1}{\beta} - 1$ $f_2 = f_3 = 0$	
		$1.5(1-\psi) \leq \alpha < 3(1-\psi)$	$f_1 = \frac{1}{\beta} - (2-\omega\beta)^4 - 9(\omega\beta-1)(\frac{2}{3}-\beta)$ $f_2 = f_3 = 0$	
		$1-\psi \leq \alpha < 1.5(1-\psi)$	For $\alpha > 1.5$, $f_1 = 2\left(\frac{1}{\beta} - 16\left(1-\frac{\omega}{3}\right)^4\right)(\frac{1}{\beta}-1)$ $f_2 = 3\beta - 2$; $f_3 = 0$ For $\alpha \leq 1.5$, $f_1 = 2\left(\frac{1.5}{1-\psi}-1\right)\left(\frac{1}{\beta}-1\right)$ $f_2 = \frac{\psi(1-16f_4^2)}{1-\alpha}$; $f_3 = 0$	

<table>
<tr><td rowspan="3"></td><td rowspan="2"></td><td></td><td>$f_4 = [1.5 - Min(1.5;\alpha)]^2$</td><td rowspan="2"></td></tr>
<tr><td>$0.75(1-\psi) \le \alpha < (1-\psi)$</td><td>$f_1 = 0$
$f_2 = 1 + 2.31(\beta - 1) - 48\left(\frac{4}{3} - \beta\right) f_4^2$
$f_3 = 3 f_4 (\beta - 1)\left(\frac{f_4}{1.81} - \frac{\alpha - 1}{1.31}\right)$
$f_4 = [1.5 - Min(1.5;\alpha)]^2$</td></tr>
<tr><td>$\psi < 1 - \frac{4\alpha}{3}$</td><td colspan="2">$K_y = 5.972 F_{tran} \frac{\beta^2}{1 - f_3}$
Where,
$f_3 = f_5\left(\frac{f_5}{1.81} + \frac{1+3\psi}{5.24}\right)$
$f_5 = \frac{9}{16}[1 + Max(-1;\psi)]^2$</td><td></td></tr>
<tr><td>③ </td><td>$1 \ge \psi \ge 0$</td><td>$K_x = \frac{4(0.425 + 1/\alpha^2)}{3\psi + 1}$</td><td rowspan="2">$C_x = 1$ for $\lambda \le 0.7$
$C_x = \frac{1}{\lambda^2 + 0.51}$ for $\lambda > 0.7$</td></tr>
<tr><td>$0 > \psi > -1$</td><td>$K_x = 4(0.425 + 1/\alpha^2)(1+\psi) - 5\psi(1 - 3.42\psi)$</td></tr>
</table>

<table>
<tr>
<td>④ </td>
<td>$1 \geq \psi \geq -1$</td>
<td colspan="2">$K_x = \left(0.425 + \frac{1}{\alpha^2}\right)\frac{3-\psi}{2}$</td>
<td rowspan="2"></td>
</tr>
<tr>
<td>⑤ </td>
<td>-</td>
<td>$\alpha \geq 1.64$</td>
<td>$K_x = 1.28$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha < 1.64$</td>
<td>$K_x = \frac{1}{\alpha^2} + 0.56 + 0.13\alpha^2$</td>
</tr>
<tr>
<td>⑥ </td>
<td>$1 \geq \psi \geq 0$</td>
<td colspan="2">$K_y = \frac{4(0.425+\alpha^2)}{(3\psi+1)\alpha^2}$</td>
<td rowspan="3">$C_y = 1$ for $\lambda \leq 0.7$
$C_y = \frac{1}{\lambda^2 + 0.51}$ for $\lambda > 0.7$</td>
</tr>
<tr>
<td></td>
<td>$0 > \psi \geq -1$</td>
<td colspan="2">$K_y = 4\left(0.425+\alpha^2\right)(1+\psi)\frac{1}{\alpha^2} - 5\psi(1-3.42\psi)\frac{1}{\alpha^2}$</td>
</tr>
<tr>
<td>⑦ </td>
<td>$1 \geq \psi \geq -1$</td>
<td colspan="2">$K_y = \left(0.425+\alpha^2\right)\frac{3-\psi}{2\alpha^2}$</td>
</tr>
</table>

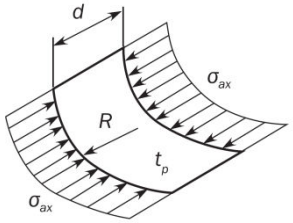
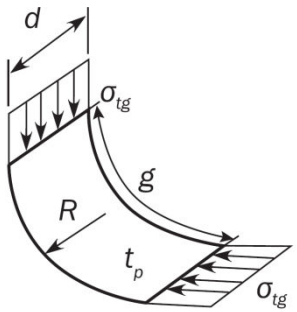
 ⑧	--	$K_y = 1 + \frac{0.56}{\alpha^2} + \frac{0.13}{\alpha^4}$	
 ⑨	—	$K_x = 6.97$	$C_x = 1$ for $\lambda \le 0.83$ $C_x = 1.13\left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2}\right)$ for $\lambda > 0.83$
 ⑩	—	$K_y = 4 + \frac{2.07}{\alpha^2} + \frac{0.67}{\alpha^4}$	$C_x = 1$ for $\lambda \le 0.83$ $C_x = 1.13\left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2}\right)$ for $\lambda > 0.83$
 ⑪	—	$\alpha \ge 4$: $K_x = 4$ $\alpha < 4$: $K_x = 4 + 2.74\left[\frac{4-\alpha}{3}\right]^4$	$C_x = 1$ for $\lambda \le 0.83$ $C_x = 1.13\left(\frac{1}{\lambda} - \frac{0.22}{\lambda^2}\right)$ for $\lambda > 0.83$
⑫	-	K_y determined as per case ②.	For $\alpha < 2$, $C_y = C_{y2}$; For $\alpha \ge 2$,

				$C_y=\left(1.06+\frac{1}{10\alpha}\right)C_{y2}$; Where, C_{y2} determined as per case ②.
⑬ 	—	$\alpha\geq 4$	$K_x=6.97$	$C_x=1$ for $\lambda\leq 0.83$ $C_x=1.13\left(\frac{1}{\lambda}-\frac{0.22}{\lambda^2}\right)$ for $\lambda>0.83$
		$\alpha<4$	$K_x=6.97+3.1\left[\frac{4-\alpha}{3}\right]^4$	
 ⑭	—	$K_y=\frac{6.97}{\alpha^2}+\frac{3.1}{\alpha^4}\left(\frac{4-1/\alpha}{3}\right)^4$		$C_\tau=1$ for $\lambda\leq$ 0.83 $C_y=1.13\left(\frac{1}{\lambda}-\frac{0.22}{\lambda^2}\right)$ for $\lambda>0.83$
 ⑮	—	$K_\tau=\sqrt{3}\left[5.34+\frac{4}{\alpha^2}\right]$		$C_\tau=1$ for $\lambda\leq 0.84$ $C_\tau=\frac{0.84}{\lambda}$ for $\lambda>0.84$

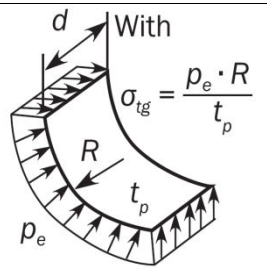
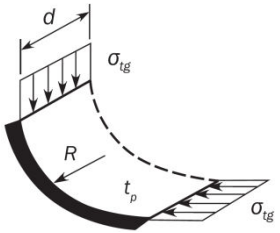
⑯ 	—	$K_\tau = \sqrt{3}\left[5.34 + Max\left[\frac{4}{\alpha^2}; \frac{7.15}{\alpha^{2.5}}\right]\right]$	
⑰ 	—	$K_\tau = K_{\tau case15} r$ $K_{\tau case15}$: K_τ according to case ⑮ *r=Opening reduction factor taken as:* $r = \left(1 - \frac{d_a}{a}\right)\left(1 - \frac{d_b}{b}\right)$ $\frac{d_a}{a} \le 0.7$ **and** $\frac{d_b}{b} \le 0.7$	
⑱ 	—	$K_\tau = \sqrt{3}\left(0.6 + \frac{4}{\alpha^2}\right)$	$C_\tau = 1$ **for** $\lambda \le 0.84$ $C_\tau = \frac{0.84}{\lambda}$ **for** $\lambda > 0.84$
⑲ 	—	$K_\tau = 8$	

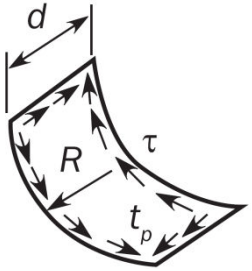
Edge boundary conditions:
- - - - - - - - - - - - Plate edge free;
———————— Plate edge simply supported;
▬▬▬▬▬▬▬▬▬▬▬▬ Plate edge clamped.
Note (1): Cases listed are general cases. Each stress component (σ_x, σ_y) is to be understood in local coordinates.

Buckling Factor and Reduction Factor for Curved Plate Panel with $R / t_p \le 2500$ Table 1.3.1

<table>
<tr><th>Case</th><th>Aspect ratio</th><th>Buckling factor K</th><th>Reduction factor C</th></tr>
<tr><td rowspan="2">1

</td><td>$\frac{d}{R} \le 0.5\sqrt{\frac{R}{t_p}}$</td><td>$K = 1 + \frac{2}{3}\frac{d^2}{Rt_p}$</td><td rowspan="2">For general application:
$C_{ax} = 1$ for $\lambda \le 0.25$
$C_{ax} = 1.233 - 0.933\lambda$ for $0.25 < \lambda \le 1$
$C_{ax} = 0.3 / \lambda^2$ for $1 < \lambda \le 1.5$
$C_{ax} = 0.2 / \lambda^2$ for $\lambda > 1.5$
For curved single fields, e.g. bilge strake, which are bounded by plane plate panels:
$C_{ax} = \frac{0.65}{\lambda^2} \le 1.0$</td></tr>
<tr><td>$\frac{d}{R} > 0.5\sqrt{\frac{R}{t_p}}$</td><td>$K = 0.267\frac{d^2}{Rt_p}\left[3 - \frac{d}{R}\sqrt{\frac{t_p}{R}}\right] \ge 0.4\frac{d^2}{Rt_p}$</td></tr>
<tr><td>2a


2b</td><td>..</td><td>$K = \frac{d}{\sqrt{Rt_p}} + 3\frac{(Rt_p)^{0.175}}{d^{0.35}}$</td><td rowspan="2">For general application:
$C_{tg} = 1$ for $\lambda \le 0.4$
$C_{tg} = 1.274 - 0.686\lambda$ for $0.4 < \lambda \le 1.2$
$C_{tg} = \frac{0.65}{\lambda^2}$ for $\lambda > 1.2$
For curved single fields, e.g. bilge strake, which are bounded by plane plate panels:
$C_{tg} = \frac{0.8}{\lambda^2} \le 1.0$</td></tr>
<tr><td>$\frac{d}{R} > 1.63\sqrt{\frac{R}{t_p}}$</td><td>$K = 0.3\frac{d^2}{R^2} + 2.25\left(\frac{R^2}{dt_p}\right)^2$</td></tr>
</table>

 p_e - External pressure, in N/mm^2.			
3 	$\frac{d}{R} \le \sqrt{\frac{R}{t_p}}$	$K = \frac{0.6d}{\sqrt{Rt_p}} + \frac{\sqrt{Rt_p}}{d} - 0.3\frac{Rt_p}{d^2}$	As in load case 2a
	$\frac{d}{R} > \sqrt{\frac{R}{t_p}}$	$K = 0.3\frac{d^2}{R^2} + 0.291\left(\frac{R^2}{dt_p}\right)^2$	
4	$\frac{d}{R} \le 8.7\sqrt{\frac{R}{t_p}}$	$K = \sqrt{3}\sqrt{28.3 + \frac{0.67d^3}{R^{1.5}t_p^{1.5}}}$	$C_\tau = 1$ for $\lambda \le 0.4$ $C_\tau = 1.274 - 0.686 \cdot \lambda$ for $0.4 < \lambda \le 1.2$

	$\frac{d}{R} > 8.7\sqrt{\frac{R}{t_p}}$	$K = \sqrt{3}\frac{0.28d^2}{R\sqrt{Rt_p}}$	$C_\tau = \frac{0.65}{\lambda^2}$ for $\lambda > 1.2$
Edge boundary conditions: - - - - - - - - - - - - Plate edge free; ————————Plate edge simply supported; ▬▬▬▬▬▬▬▬ Plate edge clamped.			

Chapter 4 Elasto-Plastic Method (EPM) for Buckling/Ultimate Strength of Stiffened Plate Panels

4.1 General requirements

4.1.1 This chapter incorporates the Elasto-Plastic Method (semi-analytical numerical method, EPM) developed by CCS for buckling/ultimate strength assessment of unstiffened/stiffened plate panels. EPM is suitable for elasto-plastic buckling assessment of stiffened plane plate panel structures.

4.1.2 Generally, EPM can further improve the buckling assessment accuracy than CFM in the following cases, where EPM can be conditionally applied as an alternative to "Method A" in CFM.

Note: The following cases can be used as the premises on which EPM can be applied as an alternative to CFM:

① Where calculation of flat bar stiffened panel structure with CFM, there is obvious decrease of buckling capacity, e.g. flat bar stiffened panel structure when plate panel's aspect ratio $\alpha \geq 6$;

② Where calculation of plate panel having the aspect ratio of $1 < \alpha < 2$ with CFM, the panel buckling capacity decreases with decreasing α;

③ Orthotropic stiffened plate panel, densely stiffened plate panel;

④ Lightweight stiffened plate panel, such as the stiffened panel structure of superstructure bulkhead and the stiffened panel structure on high-speed craft;

⑤ Other cases in which unreasonable results are obtained with CFM.

4.1.3 The calculation with EPM is performed with CCS dedicated software COMPASS-ABA-EPM.

Note: As the functions of COMPASS-ABA-EPM are expanding continuously, it should be noted that some functions may not be included yet, e.g. buckling assessment functions for reference stress distribution regarding special plate panel edge, special plate panel boundary constraint conditions and specially shaped plate panels. See the instructions of the latest version of COMPASS-ABA-EPM for details.

4.1.4 It should be implemented that the requirements in this Chapter are to be complied with when using EPM to solve the buckling/ultimate capacity of stiffened/unstiffened plate panels; for other applications, the same requirements as those for CFM are to be complied with.

4.1.5 In applying EPM for buckling assessment, 'Method A' as defined in Chapter 2 is subject to the following prescription:

"Method A" - corresponding to elasto-plastic buckling assessment results obtained with EPM.

4.2 Basic method

4.2.1 Overview of EPM

EPM is based on the elastic large-deflection plate theory and rigid-plastic analysis method. By analyzing various potential buckling failure modes of unstiffened/stiffened panel structures, EPM can identify the most critical failure mode and corresponding buckling capacity.

Main steps using EPM:

(1) Determine the type of analysis model for a plate panel and analyze various potential failure modes. See 4.2.2 of this chapter.

(2) For each failure mode, derive a stress-deflection curve by using the elastic large-deflection theory, and another edge stress ϕ-deflection ψ curve by using the rigid-plastic theory. The intersection of these two curves corresponds to the ultimate strength of the stiffened plate panel of the failure mode. See Figure 4.2.1 (1).

(3) Among all potential failure modes of the unstiffened panel model, longitudinally stiffened panel model and orthotropic stiffened panel analysis model respectively, the critical failure mode corresponding to the intersection with the minimum edge stress. And this identified minimum edge stress is defined as the actual ultimate capacity of the stiffened panel. See Figure 4.2.1 (2).

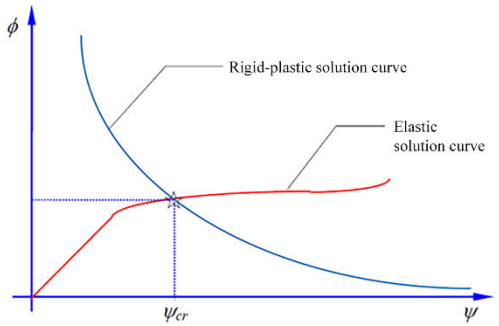


Figure 4.2.1 (1) Elasto-plastic Buckling Calculation Analysis Curve for One Buckling Failure Mode

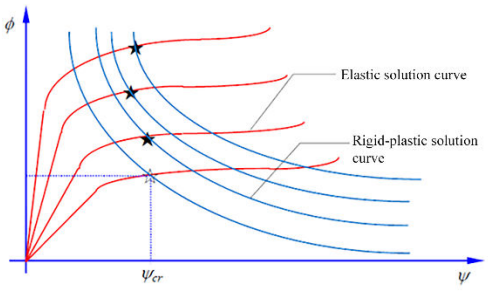


Figure 4.2.1 (2) Elasto-plastic Buckling Calculation Analysis Curve for Various Buckling Failure Modes

4.2.2 Buckling failure mode analysis

(1) Stiffened panel structures can be idealized as unstiffened panel models (UP), longitudinally stiffened panel models (SP) or orthotropic stiffened panel models (OSP).

(2) For such three types of idealized analysis models, various potential failure modes are to be considered respectively. See Table 4.2.2. Meanwhile, theoretical analysis models corresponding to each failure mode are listed in the table. See 4.3 of this chapter for a brief introduction to each of the theoretical analysis models.

Buckling Failure Modes and Theoretical Analysis Models for Stiffened Plate Panels Table 4.2.2

Analytical model	Failure mode	Theoretical analysis model	
		Description	S/N
Unstiffened plate panel （UP）	Elastic buckling of unstiffened plate panels (dominated by transverse loads)	Elastic buckling of stiffened panels	⑤
	Ultimate strength of unstiffened plate panels	Ultimate strength of unstiffened plate panels	①
	Yield	Von Mises yield criterion	⑥
Longitudinal stiffened plate panel （SP）	Overall elastic buckling	Elastic buckling of stiffened panels	⑤
	Longitudinal stiffened plate panel Overall buckling	Overall buckling of stiffened panels	②
	Elastic buckling of unstiffened plate panels (dominated by transverse loads)	Elastic buckling of stiffened panels	⑤
	Ultimate strength of unstiffened plate panels (dominated by transverse loads)	Ultimate strength of unstiffened plate panels	①
	Flexural buckling of stiffeners	Flexural buckling of stiffeners	③
	Torsional buckling of stiffeners	Torsional/tripping buckling of stiffeners	④
	Full yield	Von Mises yield criterion	⑥
Orthogonally stiffened plate panels （OSP）	Orthogonally stiffened plate panels Overall elastic buckling	Elastic buckling of stiffened panels	⑤
	Orthogonally stiffened plate panels	Overall buckling of stiffened panels	②

	Overall buckling		
	Longitudinally stiffened sub-plate panel Overall elastic buckling	Elastic buckling of stiffened panels	⑤
	Longitudinally stiffened sub-plate panel Overall buckling	Overall buckling of stiffened panels	②
	Flexural buckling of stiffeners	Flexural buckling of stiffeners	③
	Torsional buckling of stiffeners	Torsional/tripping buckling of stiffeners	④
	Full yield	Von Mises yield criterion	⑥

4.3 Theoretical analysis model

Note: This section mainly describes principles. Some formulae (equations) and symbols descriptions in diagrams are omitted or simplified in the provisions of this section.

4.3.1 Theoretical analysis model ①: ultimate strength of unstiffened plate panels

(1) Basic assumptions and calculation results:

(a) Initial deflection function for a single component:

$$w_0 = A_{0ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b}$$

(b) Total deformation function due to in-plane bi-axial and lateral pressures:

$$w = A_{kl} \sin\frac{k\pi x}{a} \sin\frac{l\pi y}{b}$$

(c) Deflection function due to edge shear:

$$w_s = \sum_{k=1}^{3} A_{skk} \sin\frac{k\pi x}{a} \sin\frac{k\pi y}{b}$$

(d) The longitudinal and transverse welding residual stresses of side plate panels are distributed as follows; see Figure 4.3.1 (1):

$$\sigma_{rx} = \begin{cases} \sigma_{rtx}(=\sigma_0), & y \in (0, b_t) \\ \sigma_{rcx}, & y \in (b_t, b-b_t) \\ \sigma_{rtx}(=\sigma_0), & y \in (b-b_t, b) \end{cases}, \quad \sigma_{ry} = \begin{cases} \sigma_{rty}(=\sigma_0), & x \in (0, a_t) \\ \sigma_{rcy}, & x \in (a_t, a-a_t) \\ \sigma_{rty}(=\sigma_0), & x \in (a-a_t, a) \end{cases}$$

Where: σ_{rtx} and σ_{rty} - longitudinal and transverse tensile residual stresses respectively;

σ_{rcx} and σ_{rcy} - longitudinal and transverse compressive residual stresses respectively;

For mild steel: $\sigma_{rtx} = \sigma_{rty} = \sigma_0$

For high strength steel: $\sigma_{rtx} = \sigma_{rty} = 0.8\sigma_0$

σ_0 - yield stress of panel

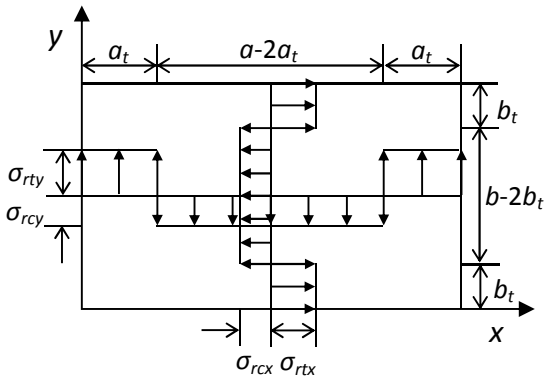


Figure 4.3.1 (1) Distribution of Welding Residual Stress of Plate Panel

(e) Two kinematically admissible failure modes depending on aspect ratio

(f) A set of deformation modes (*k, l*) can be obtained by varying *i* from 1 to M and *j* from 1 to N, corresponding to the minimum intersection point between elastic large deflection solution and rigid-plastic solution. The ultimate strength of a panel is defined as the minimum intersection point in the load-deformation plane formed by all response functions for the individual components of the initial deflection function.

(2) Elastic large deflection analysis

(a) For the simply supported rectangular plate panels under in-plane bi-axial stresses and lateral pressures, the compatibility equation of elastic large deflection panel taking into account initial deflection is as follows:

$$\nabla^4 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2 + \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}\right]$$

Boundary conditions:

$$\int_0^{b_t}\sigma_x dy=(\sigma_{ax}+\sigma_{rtx})b_t,\quad \int_0^{b_t}\sigma_x\left(y-\frac{b}{2}\right)dy+\int_{b-b_t}^{b}\sigma_x\left(y-\frac{b}{2}\right)dy=0,$$

$$\int_{b_t}^{b-b_t}\sigma_x dy=(\sigma_{ax}+\sigma_{rcx})(b-2b_t),\quad \int_{b_t}^{b-b_t}\sigma_x\left(y-\frac{b}{2}\right)dy=0$$

$$\int_0^{a_t}\sigma_y dx=(\sigma_{ay}+\sigma_{rty})a_t,\quad \int_0^{a_t}\sigma_y\left(x-\frac{a}{2}\right)dx+\int_{a-a_t}^{a}\sigma_y\left(x-\frac{a}{2}\right)dx=0,$$

$$\int_{a_t}^{a-a_t}\sigma_y dx=(\sigma_{ay}+\sigma_{rcy})(a-2a_t),\quad \int_{a_t}^{a-a_t}\sigma_y\left(x-\frac{a}{2}\right)dx=0$$

$$\tau_{xy}\big|_{\substack{x=0,a\\y=0,b}}=0,\quad \tau_{xy}\big|_{\substack{x=a/2\\y=0/2}}=0$$

Where, F - stress function;

E - elasticity modulus of material.

(b) Potential energy equation:

$$E_C+E_B+W_\sigma+W_p=0$$

Where: E_c - compressive strain energy of plate panel;

E_B - bending strain energy of plate panel;

W_σ - work under bi-axial pressure of plate panel;

W_p - work under longitudinal pressure of plate panel;

(c) Equilibrium equations for elastic large deflection analysis of plate panels taking into account shear stress effects:

$$\frac{D}{t}\nabla^4 w_s=\frac{\partial^2 F}{\partial y^2}\left(\frac{\partial^2 w_s}{\partial x^2}+\frac{\partial^2 w_0}{\partial x^2}\right)+\frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w_s}{\partial y^2}+\frac{\partial^2 w_0}{\partial y^2}\right)-2\frac{\partial^2 F}{\partial x\partial y}\left(\frac{\partial^2 w_s}{\partial x\partial y}+\frac{\partial^2 w_0}{\partial x\partial y}\right)$$

Compatibility equation:

$$\frac{1}{E}\nabla^4 F=\left(\frac{\partial^2 w_s}{\partial x\partial y}\right)^2-\frac{\partial^2 w_s}{\partial x^2}\frac{\partial^2 w_s}{\partial y^2}+2\frac{\partial^2 w_s}{\partial x\partial y}\frac{\partial^2 w_0}{\partial x\partial y}-\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_s}{\partial y^2}-\frac{\partial^2 w_s}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}$$

(3) Rigid-plastic solution

(a) Establish a rigid-plastic solution for the assumed failure mode based on the energy principle between internal energy and external work.

(b) Two kinematically admissible failure modes. See Figure 4.3.1 (2):

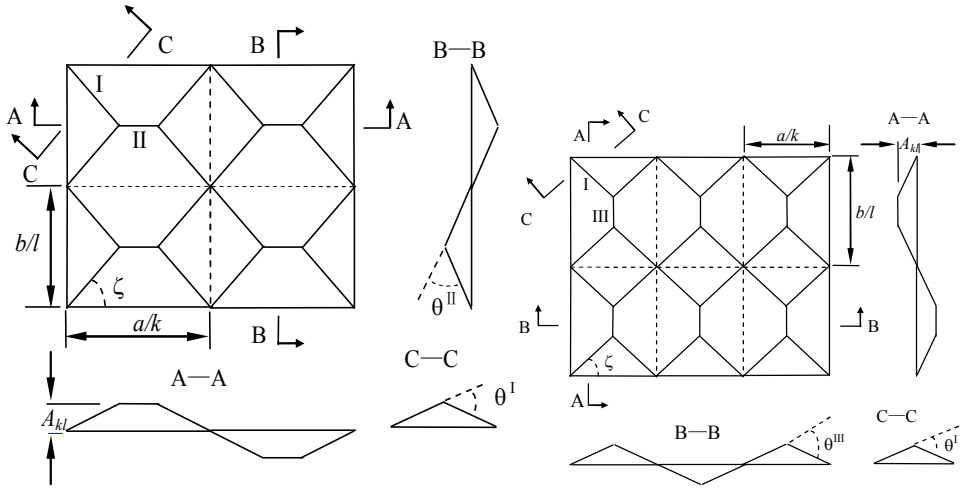


Figure 4.3.1 (2) Two Plastic Failure Modes of Panel

(c) By means of formula calculus, the plastic bending moment at the plastic hinge line is obtained as follows:

$$M_p(\zeta)=\sigma_{1t}\frac{c^2}{2}-\sigma_{1c}\frac{(t-c)^2}{2}=m_p(\zeta)\frac{\sigma_0 t^2}{4}=\frac{t^2}{2}\frac{\sigma_1^2-\sigma_{1t}\sigma_{1c}}{\sigma_{1t}-\sigma_{1c}}=\frac{t^2}{2}\frac{\sigma_0^2+\sigma_1^2-\sigma_2^2-3\tau^2}{\sqrt{4\sigma_0^2-3\sigma_2^2-12\tau^2}}$$

For hinge lines I, II and III at 45° and 90° respectively, the plastic bending moments are obtained as follows:

$$m_{\mathrm{I}}=m_p(45^o)=\frac{4+8(\phi_x+\phi_y)\phi_{xy}-3(\phi_y-\phi_x)^2}{\sqrt{16-3(\phi_x+\phi_y-2\phi_{xy})^2-12(\phi_y-\phi_x)^2}}$$

$$m_{\mathrm{II}}=m_p(0^o)=\frac{2(1+\phi_y^2-\phi_x^2-3\phi_{xy}^2)}{\sqrt{4-3\phi_x^2-12\phi_{xy}^2}}$$

$$m_{\mathrm{III}}=m_p(90^o)=\frac{2(1+\phi_x^2-\phi_y^2-3\phi_{xy}^2)}{\sqrt{4-3\phi_y^2-12\phi_{xy}^2}}$$

4.3.2 Theoretical analysis model ②: overall buckling of stiffened panels

(1) Basic assumptions

(a) In the overall buckling failure mode, the stiffened plate panel behaves more like a plane plate. Therefore, for this failure mode, the stiffened panel can be transformed into an equivalent anisotropic plane plate and then analyzed with the method for the ultimate strength of a plane plate.

(b) In addition to the assumptions regarding residual stresses, the basic assumptions in 4.3.1 of this chapter also apply. Residual stress distribution in T-shaped stiffeners is shown in Figure 4.3.2 (1), and that of flat bar stiffeners is shown in Figure 4.3.2 (2). The welding residual stress distribution of the angle profile and bulb profile is similar to that of the T-shaped profile.

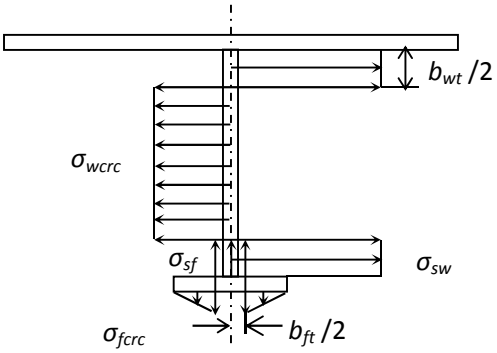


Figure 4.3.2 (1) Residual Stress Distribution of T-Bar Stiffener

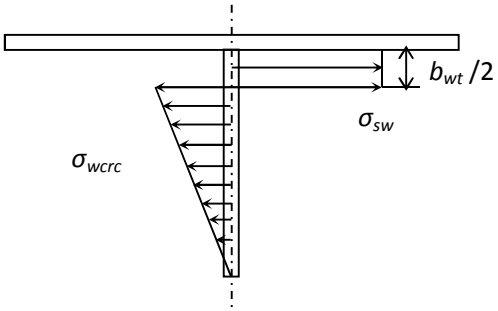


Figure 4.3.2 (2) Residual Stress Distribution of Flat Bar Stiffener

(2) Elastic large deflection analysis

The compatibility equation for orthotropic plate with initial deflection which is subject to elastic large deflection:

$$\frac{1}{E_y}\frac{\partial^4 F}{\partial x^4}+\left(\frac{1}{G}-2\frac{v_x}{E_x}\right)\frac{\partial^4 F}{\partial x^2\partial y^2}+\frac{1}{E_x}\frac{\partial^4 F}{\partial y^4}$$
$$=\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2+\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}$$

Compressive strain energy E_{IP} and bending strain energy E_{IB} of orthotropic plate:

$$E_{IP}=\frac{t}{2}\int_0^a\int_0^b \sigma_x\varepsilon_x+\sigma_y\varepsilon_y\,dxdy$$

$$E_{IB}=\int_0^a\int_0^b \frac{D_x}{2}\left[\frac{\partial^2(w-w_0)}{\partial x^2}\right]^2+\frac{D_y}{2}\left[\frac{\partial^2(w-w_0)}{\partial y^2}\right]^2$$
$$+\frac{D_x v_y+D_y v_x}{2}\left[\frac{\partial^2(w-w_0)}{\partial x^2}\right]\left[\frac{\partial^2(w-w_0)}{\partial y^2}\right]+\mathrm{H}\left[\frac{\partial^2(w-w_0)}{\partial x\partial y}\right]^2 dxdy$$

External work of orthotropic plate:

$$W_p = \int_0^a \int_0^b q(w - w_0)dxdy + \bar{u}\sigma_{xav}(1 + N_{bx}A_{fx} + N_{bx}A_{wx})bt + \bar{v}\sigma_{yav}(1 + N_{by}A_{fy} + N_{by}A_{wy})at$$

By applying the energy principle $E_{IP} + E_{IB} + W_p = 0$, the following equation for elastic large deflection deformation can be derived:

$$S\psi_{kl}^3 + P\psi_{kl} + Q = 0$$

(3) Rigid-plastic solution

Internal work W_I and external work W_E of orthotropic plate:

$$W_I = \sum_{n=1}^{N} \int_{l_n} M_{pn}\delta\theta_n dl_n$$

$$W_E = \sum_{n=1}^{N} \int_{l_n} N_n w_n \delta\theta_n dl_n$$

Using the upper bound theorem of plasticity, the following rigid-plastic solution can be derived:

$$R\psi_{kl} = T$$

4.3.3 Theoretical analysis model ③: flexural buckling of stiffeners

(1) This failure mode corresponds to a beam-column type of failure due to buckling of the attached plating. For this mode, longitudinal and transverse stiffeners are stronger, so the buckling occurs first at the plate between stiffeners, the degraded load-carrying capacity can be represented by the effective attached plating width, the longitudinal stiffeners and effective attached plating fail in a beam-column buckling mode, and the compressive ultimate strength of the stiffened plate panels can be expressed as:

$$\phi = \frac{\phi_p \sigma_{0p} + \phi_{wf} A_{wf}(N_{bx} + 1)\sigma_{0wf}}{1 + A_{wf}(N_{bx} + 1)}$$

Where: ϕ_p and ϕ_{wf} are the ultimate strengths for buckling of plate between stiffeners and of stiffeners.

ϕ_p can be calculated by the above-mentioned ultimate strength method for plane plate, while ϕ_{wf} can be solved with reference to elastic and rigid-plastic beam-column theories.

(2) Rigid-plastic solution of beam and column:

$$\psi_p = \frac{m_p\left(\frac{1}{c}+\frac{1}{1-c}\right)-\frac{\phi_b\alpha^2\beta^2}{2}}{\phi_x\left(A_w+A_f+A_e\right)\left(\frac{1}{c}+\frac{1}{1-c}\right)}$$

ϕ_{wf} corresponds to ϕ_x at the intersection point between elastic solution and rigid-plastic solution of beam and column.

4.3.4 Theoretical analysis model ④: torsional/roll buckling of stiffeners

(1) This failure mode corresponds to the type of stiffeners with a greater depth-thickness ratio or weaker stiffness of the stiffener face plate. In this mode, torsion or transverse titling occurs on the stiffeners. For this mode, longitudinal stiffeners are not sufficiently rigid against torsion and fail in a tripping manner after the plate between the stiffeners is bucked.

(2) Elastic solution for stiffened web:

$$S\varphi^3 + P\varphi + Q = 0$$

Where: $S = \frac{3J_1k^2}{4\alpha^2}$;

$$P = \frac{1}{6(1-v^2)}\left[\frac{k^2\gamma^2}{\alpha^2}J_3 + 2(1-v)\frac{\gamma^4}{\pi^2}J_4\right] - \frac{J_1 i^2(2+\delta_{ik})}{4\alpha^2}\varphi_{0i}^2 + \frac{2\beta^2[J_2-(A_w+A_f)h]}{\pi^2}\phi_x$$

$$Q = \frac{1}{6(1-v^2)}\left[\frac{k^2\gamma^2}{\alpha^2}J_3 + 2(1-v)\frac{\gamma^4}{\pi^2}J_4\right]\varphi_{0i}\delta_{ik}$$

$$J_1 = \left(\lambda_w-\lambda_f\right)\left(\frac{\gamma_w^5}{5}-\frac{2h\gamma_w^3}{3}+h^2\gamma_w\right)+\lambda_f\left[\frac{\left(\gamma_w+\gamma_f\right)^5}{5}-\frac{2h\left(\gamma_w+\gamma_f\right)^3}{3}+h^2\left(\gamma_w+\gamma_f\right)\right]$$

$$J_2 = \left(\lambda_w-\lambda_f\right)\left(\frac{\gamma_w^3}{3}-h\gamma_w\right)+\lambda_f\left[\frac{\left(\gamma_w+\gamma_f\right)^3}{3}-h\left(\gamma_w+\gamma_f\right)\right]$$

$$J_3 = \frac{\lambda_w^3\gamma_w^3+\lambda_f^3\left[\left(\gamma_w+\gamma_f\right)^3-\gamma_w^3\right]}{3}$$

$$J_4 = \lambda_w^3\gamma_w+\lambda_f^3\gamma_f$$

(3) Rigid-plastic solution for stiffened web:

$$R\varphi = T$$

Where: $R = \phi_x\left(\gamma_w^2\lambda_w + \gamma_f^2\lambda_f + 2\gamma_w\gamma_f\lambda_f\right)$;

$T = m_p\left(\gamma_w\lambda_w^2 + \gamma_f\lambda_f^2\right)\gamma$。

$T = m_p\left(\gamma_w\lambda_w^2 + \gamma_f\lambda_f^2\right)\gamma$.

ϕ_{wf} stands for the minimum ϕ_x corresponding to the intersection points of all elastic and rigid-plastic $\varphi_k - \phi_x$ curves.

4.3.5 Theoretical analysis model ⑤: elastic buckling of stiffened panel

(1) This failure mode corresponds to relatively weak longitudinal and transverse stiffeners. In this mode, the stiffened plate panels can be regarded as orthotropic plates.

(2) By introducing the boundary elastic angular displacement mode function, as well as the strain energy, angular constrained elastic potential energy and external work of orthotropic plate, the eigenvalue by using the variational equation can be derived:

$$\lambda = \frac{\bar{w}_1 + 2\pi^2\bar{w}_2}{\bar{w}_0}$$

(3) The elastic buckling stress of the stiffened plate panel corresponds to its minimum eigenvalue (for different half-wave number combinations m, n), i.e.:

$$\sigma_E = \min_{(m,n)} \lambda\sigma_a$$

4.3.6 Theoretical analysis model ⑥: Von Mises yield criterion

(1) For this failure mode, generally one side of the stiffened plate panel is in tension and the other side thereof is in compression or both sides thereof are in compression, resulting in overall yielding.

(2) Under the limit state, the structure is to meet Von Mises yield criterion:

$$\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 = \sigma_0^2$$

Where: σ_x and σ_y - longitudinal and transverse stresses of stiffened plate panel respectively;

τ_{xy} - in-plane shear stress of plate panel;

σ_0 - yield strength of material.

4.4 EPM calculation software COMPASS-ABA-EPM

4.4.1 This software is an advanced buckling analysis program for hull plate panel structure developed by CCS according to relevant theories of EPM in this chapter.

4.4.2 The software reasonably takes into account non-linear geometrical behaviour, inelastic material behaviour, initial imperfection and also residual stresses etc. The stresses of stiffened plate panels include longitudinal stress, transverse stress, edge shear stress and lateral pressure. The software can effectively calculate the buckling and ultimate capacities of stiffened plate panels.

4.4.3 The software generally consists of three parts: input module, output module and core computing module. The main interface and graphic display for failure mode are as shown in Figures 4.4.3 (1) and 4.4.3 (2).

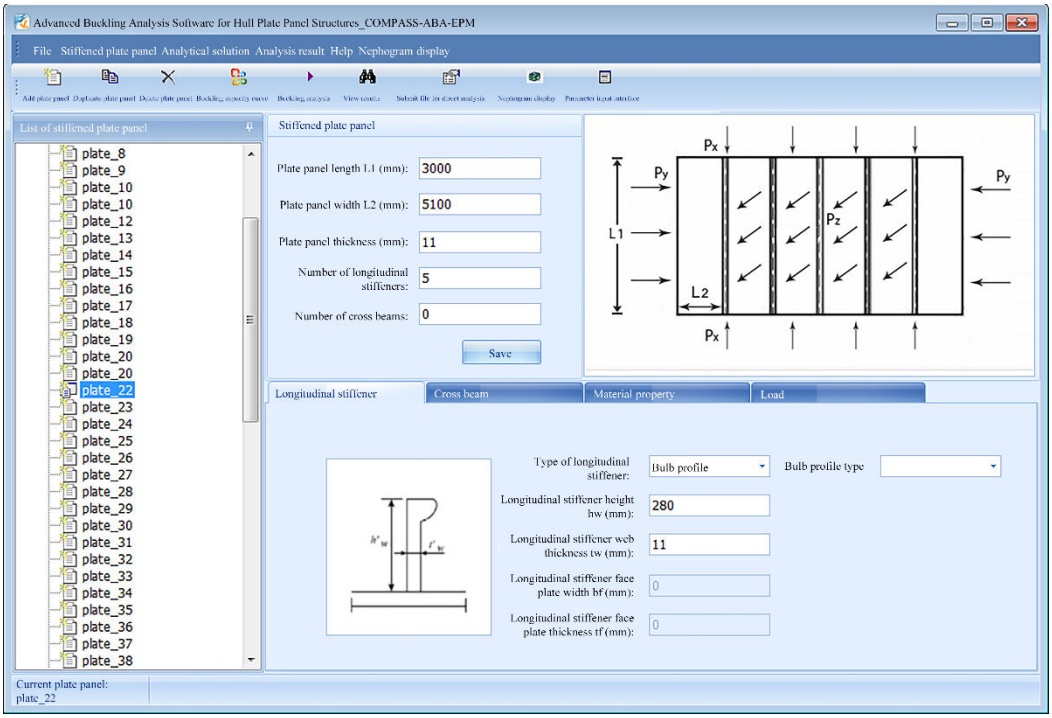


Figure 4.4.3 (1) Main Interface of COMPASS-ABA-EPM Software

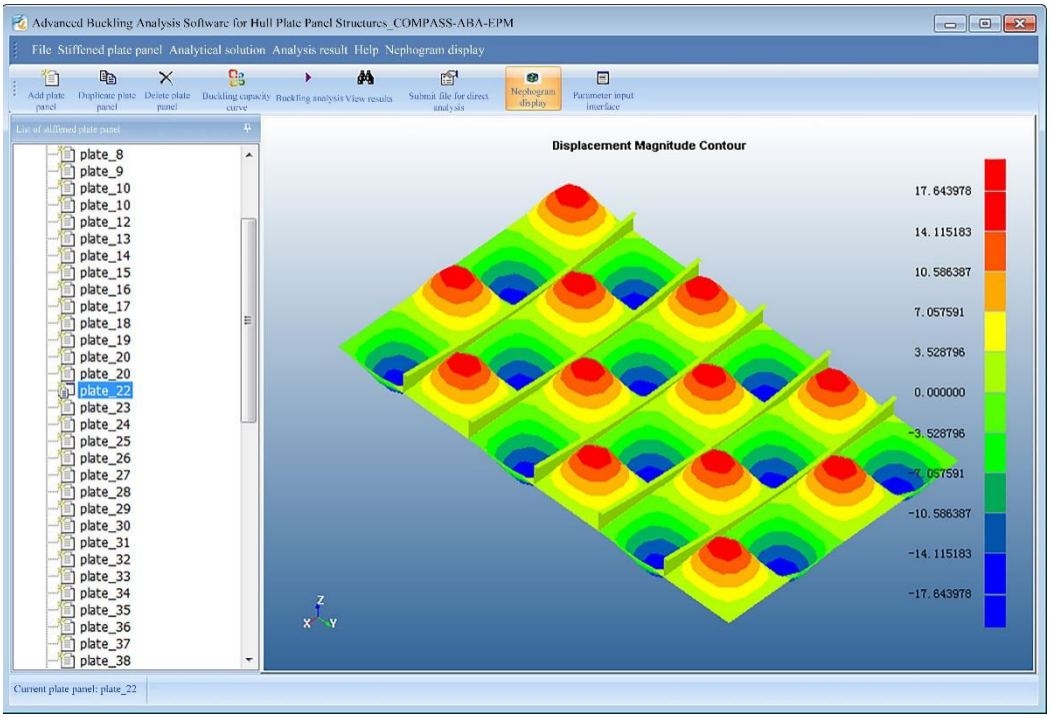


Figure 4.4.3 (2) Interface to show deformation with COMPASS-ABA-EPM Software

4.4.4 The software allows for input of plate panel information and load cases via the interface, or import of the above input data from external files.

4.4.5 The CCS ore carrier direct calculation software provides interface files that directly integrate with this software, enabling direct invocation for buckling assessments of relevant plate panels and load cases.

4.4.6 Detailed instructions on using the COMPASS-ABA-EPM software refer to its user manual.

Chapter 5 Non-Linear Finite Element Method (NLFEM) for Buckling/Ultimate Strength of Stiffened Plate Panels

5.1 General requirements

5.1.1 This chapter provides recommendations on non-linear finite element analysis of stiffened plate panels, including modelling method, boundary conditions, initial imperfection definitions and solution procedures.

Note: Although NLFEM is suitable for calculating the ultimate capacity of various structure configurations, it is recommended that it's only used in case that CFM and EPM do not apply and a more accurate assessment of individual cases is required because NLFEM is time-consuming and may sometimes bring about unstable solutions (e.g. convergence problem).

5.1.2 For the ultimate strength assessment of ship structures, both non-linear geometrical behaviour and inelastic material behaviour are to be considered.

5.1.3 Generally, the non-linear analysis process in this chapter is divided into two steps: First, the linear eigenvalues are analyzed, and the analyzed results are used as local initial imperfection data for input of non-linear analysis; second, non-linear analysis is performed to finally determine the maximum load that the plate panel can carry.

5.1.4 General-purpose non-linear finite element analysis software is preferred. Where not general-purpose software is to be used, supporting technical materials are also to be submitted to substantiate the validity of the software.

5.1.5 Thickness reduction and stress correction of the calculated structure are to be performed in accordance with the relevant requirements in Chapter 1.

5.1.6 For all analyses, the arc-length incremental solution method is to be used, so that the extreme points on the equilibrium path during the structural deformation can be traced.

5.1.7 Relevant calculation and verification reports are to be submitted as follows:

(1) The load at which the average membrane stress of elements reaches the yield stress, and the corresponding stress contour and deformation plots;

(2) Ultimate capacity: the maximum load that the plate panel can carry;

(3) Stress distribution and deformation plots under ultimate capacity;

(4) Documents to prove that the load-shortening curve stands for the stable response.

Note: For non-linear calculation of combined stress, the calculation can be deemed as stable as long as the load-shortening curve at a point of C2 or C4 features a smooth transition.

5.1.8 For non-linear finite element analysis performed in accordance with the requirements of this chapter, once the analyzed results pass the verification, the ultimate capacity derived can generally serve as an alternative to the corresponding values obtained with other methods.

Note: The verification of non-linear finite element calculation is to be based on certain experiences in non-linear finite element analysis.

5.1.9 Relevant requirements in this chapter apply to stiffened plate panels of regular shapes (rectangular uniaxial stiffened plate panel). For stiffened plate panels with irregular geometry or non-uniaxial arrangements, relevant requirements are to be discussed and considered with CCS.

5.1.10 If any method different from that in this chapter is used for non-linear analysis, further documents are to be provided as a reference for verification of results.

5.2 Model idealization

5.2.1 **Model extent:** To prevent the model from local damage on the plate panel boundary during calculation to affect the accuracy of calculation and analysis, the model is to cover the length of 3 transverse frames at the direction of stiffeners (longitudinal direction of plate panel). Generally, such length is set as 1/2+1+1+1/2 transverse frame spacings. At the transverse direction, the model is to contain 5 stiffeners, i.e. a total width of 6 stiffener spacings. See Figure 5.2.1.

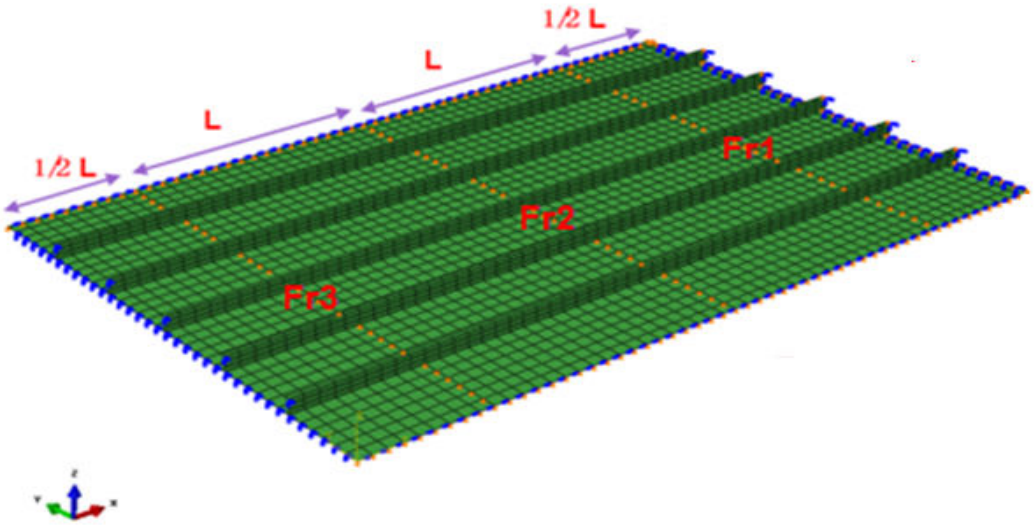


Figure 5.2.1 Range of Non-linear Finite Element Model for Stiffened Panel

5.2.2 **Material behaviour:** A bi-linear material model including the effect of strain hardening is to be used in analysis. The stress-strain curve is shown in Figure 5.2.2, and the parameters of materials to be used are:

Young's Modulus, E (N/mm^2): 206000

Poisson's ratio, ν: 0.3

Yield stress (N/mm^2): 235, 315, …

Strain hardening parameter, E_r (N/mm^2): 1000

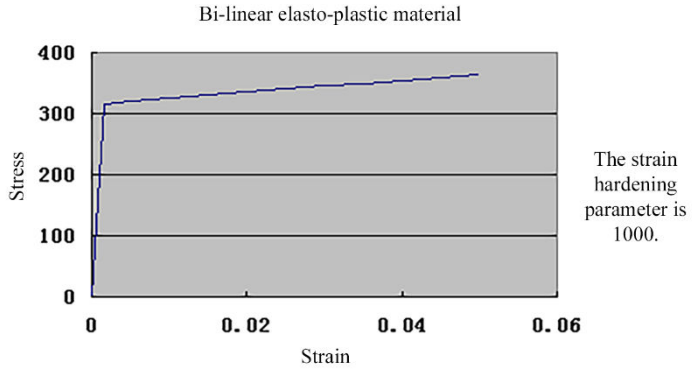


Figure 5.2.2 Stress-strain Relation of Material

5.2.3 **Coordinate system:** The model is to be based on a right-hand Cartesian coordinate system with x and y axes positioned in the plane of the plate panel and z axis normal to that plane and along the web of the stiffened panel. X axis is parallel to the stiffeners. See Figure 5.2.3.

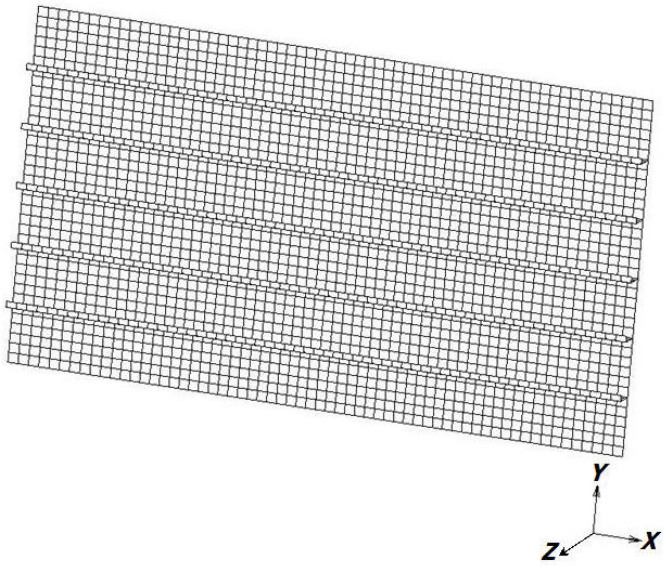


Figure 5.2.3 Coordinate System of Finite Element Model

5.3 Elements and meshes

5.3.1 Plates, stiffener webs and flanges are to be modeled as shell elements. The analysis can be performed with 4-node shell elements.

5.3.2 Modeling for shell elements of plate panel

(1) The finite element model of shell (elements) is to be established on the middle plane of plate and stiffener;

(2) Scantling for stiffener model is shown in Figure 5.3.2, and calculated as follows:

$$h'_w = h_w + {}^{t_p}\!/_{2} + {}^{t_f}\!/_{2}$$

$$b'_f = b_f - {}^{t_w}\!/_{2}$$

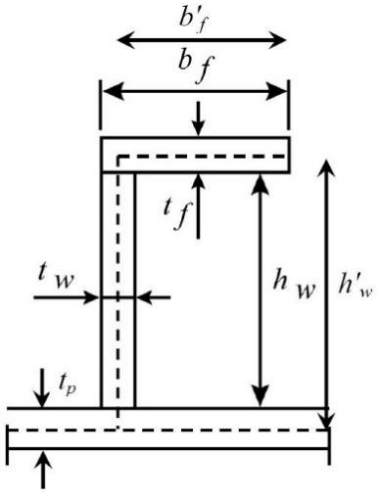


Figure 5.3.2 Equivalent Stiffener Scantling

5.3.3 Modeling to transform bulb profile into angle profile is subject to 3.3.3 (3) in Chapter 3.

5.3.4 Meshes should be sufficiently fine so that the stress distribution during local deformation and buckling development can be described accurately. In general, the following mesh densities are to be used:

- **Plate:** Mesh density between stiffeners is to ensure that the local buckling deformation of the plate panel (usually a first-order eigenvalue waveform) can be reflected accurately and that the elements are approximately square. Based on the principles above, for typical ship scantlings, the typical length of plate elements can generally be set as s/6, with s standing for the stiffener spacing;
- **Stiffener web:** A minimum of 3 elements are to be set over the depth of the web, and the elements are to be as square as possible.
- **Stiffener face plate:** For the angle profile and bulb profile, at least 1 element is to be provided over the width of the face plate; for the T-shaped profile, at least 2 elements are to be provided.

5.4 Boundary conditions and loads

5.4.1 Boundary conditions

According to the edge and corner nodes labels shown in Figure 5.4.1, the boundary conditions of the calculation model are to be set according to the requirements in Table 5.4.1.

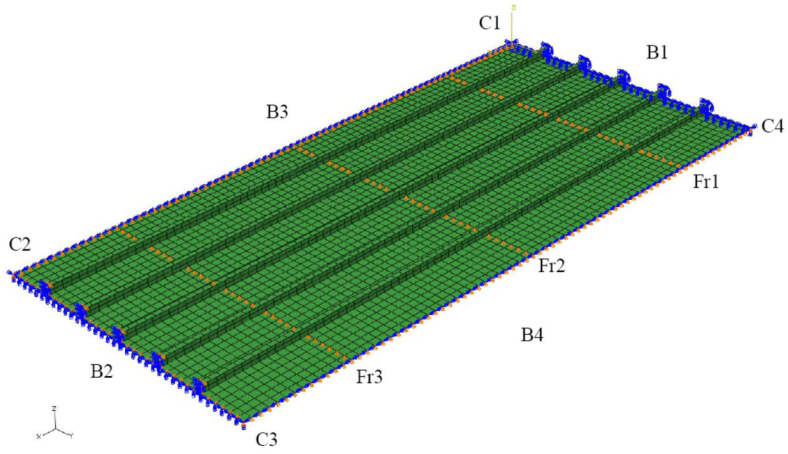


Figure 5.4.1 Edge and Corner Nodes Labels in Stiffened Panel Model

Requirements for Model Boundary Setting Table 5.4.1

Boundary area	Constraint
Edge B1, excluding corner node	Constrain displacement at x axis Constrain rotation on y and z axes
Edge B2, excluding corner node	Restrict displacement at x axis, to comply with C2 Constrain rotation on y and z axes
Edge B3, excluding corner node	Restrict to remain in line between C1 and C2 Constrain displacement at z axis Constrain rotation on x axis
Edge B4, excluding corner node	Restrict to remain linear displacement between C3 and C4 Constrain displacement at z axis Constrain rotation on x axis
Corner point C1	Constrain displacement at x, y and z axes
Corner point C2	Constrain displacement at z axis Restrict displacement at y axis to be a function of rotation and displacement for C4 (so that side B3 remains straight)
Corner point C3	Constrain displacement at z axis Restrict displacement at y axis to be a function of rotation and displacement for C4 (so that side B4 remains straight)
Corner point C4	Constrain displacement at x and z axes
Dotted lines at Fr1, Fr2 and Fr3	Constrain displacement at z axis
Intersection line between plate and stiffener	Constrain displacement at z axis Note: This boundary condition is only to be applied for the calculation of local elastic buckling mode of plate panel.

5.4.2 Applying loads

(1) For the non-linear calculation, under the combined action of in-plane load and lateral pressure on the plate panel, the load is to be applied in two steps: 1, the lateral pressure increases to the given value; 2, the in-plane load increases proportionally while the lateral pressure remains unchanged.

Note: The load calculation formulae in this section do not apply to the case of meshing with 8-node shell elements.

(2) Axial force

The in-plane axial force is applied as a point load to the corner point C2. See Figure 5.4.2 (1). This node is the main displacement node of side B1. The constraint equations ensure that the load must be distributed along the edge so that the edge can be kept straight.

The concentrated force is:

$$F_{xx}^{C1} = \sigma_x A_x = \sigma_x \left(\left(n_{stiff} + 1 \right) s t_p + n_{stiff} \left(h_w t_w + b_f t_f \right) \right)$$

Where: σ_x - desired axial stress;

n_{stiff} - number of stiffeners, taken as $n_{stiff} = 5$.

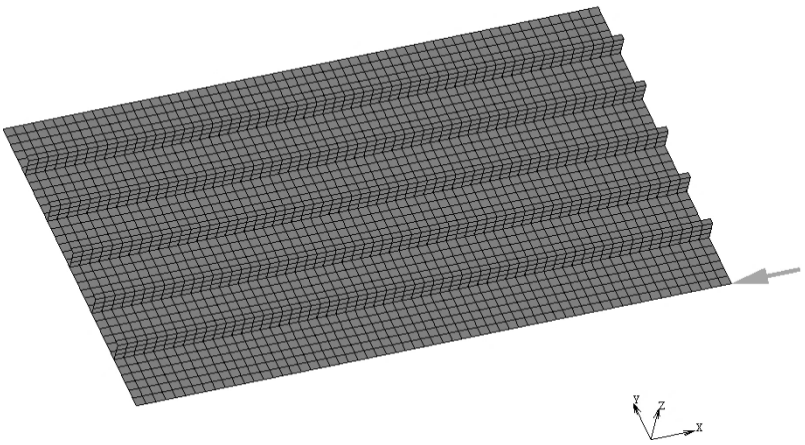


Figure 5.4.2 (1) Application of Axial Force

(3) Transverse force

In-plane transverse force is applied as the distributed load to all nodes of sides B3 and B4. See Figure 5.4.2 (2). Except for the four corner points C1-C4, the loads applied on each node at sides B3 and B4 are to be taken as:

$$F_{yy}^{B3} = \frac{\sigma_y A_y}{n_x} = \frac{\sigma_y n_{fr} l t_p}{n_x}$$

$$F_{yy}^{B4} = -\frac{\sigma_y A_y}{n_x} = -\frac{\sigma_y n_{fr} l t_p}{n_x}$$

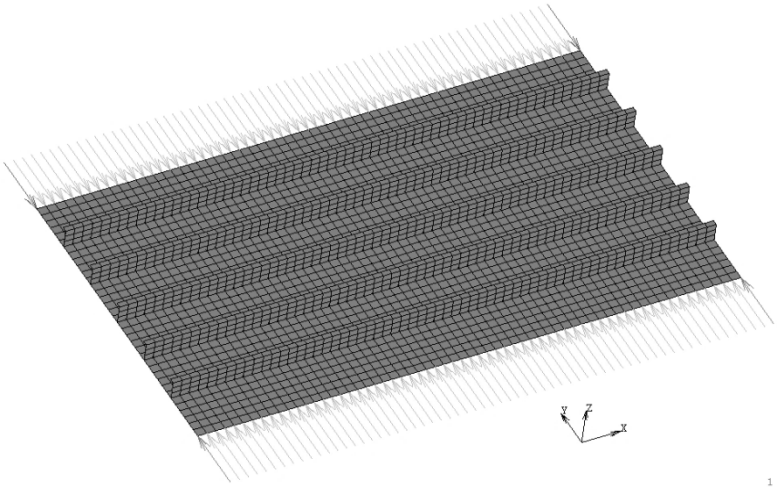


Figure 5.4.2 (2) Application of Transverse Force

Where: n_x - number of elements along the edge parallel to x axis;

n_{fr} - number of frames in the model, n_{fr}=3. This force is to be applied to all nodes except for the 4 corner points C1-C4.

At the four corner points C1-C4, the applied load equals the corresponding value calculated with the formulae above and divided by 2.

(4) Shear stress

The in-plane shear force is assumed to be taken only by the plate. Shear force is applied in the same way as transverse force, i.e. point load is applied to nodes at the edge of the plate. See Figure 5.4.2 (3). Except for the four corner points C1-C4, the load applied is calculated by:

$$F_{yx}^{B1} = -\frac{\tau_{xy} A_{x-plate}}{n_y} = -\frac{\tau_{xy}\left(n_{stiff}+1\right) s t_p}{n_y} \quad ;$$

$$F_{yx}^{B2} = \frac{\tau_{xy} A_{x-plate}}{n_y} = \frac{\tau_{xy}\left(n_{stiff}+1\right) s t_p}{n_y} \quad ;$$

$$F_{xy}^{B3} = -\frac{\tau_{xy} A_y}{n_x} = -\frac{\tau_{xy} n_{fr} l t_p}{n_x} \quad ;$$

$$F_{xy}^{B4} = \frac{\tau_{xy} A_y}{n_x} = \frac{\tau_{xy} n_{fi} l t_p}{n_x}$$

Where: n_i - number of elements along the edge parallel to i axis;

F_{ij} - force along i axis, acting on edge nodes whose normal direction is parallel to j axis

At the four corner points C1-C4, the applied load equals the corresponding value calculated with the formulae above and divided by 2.

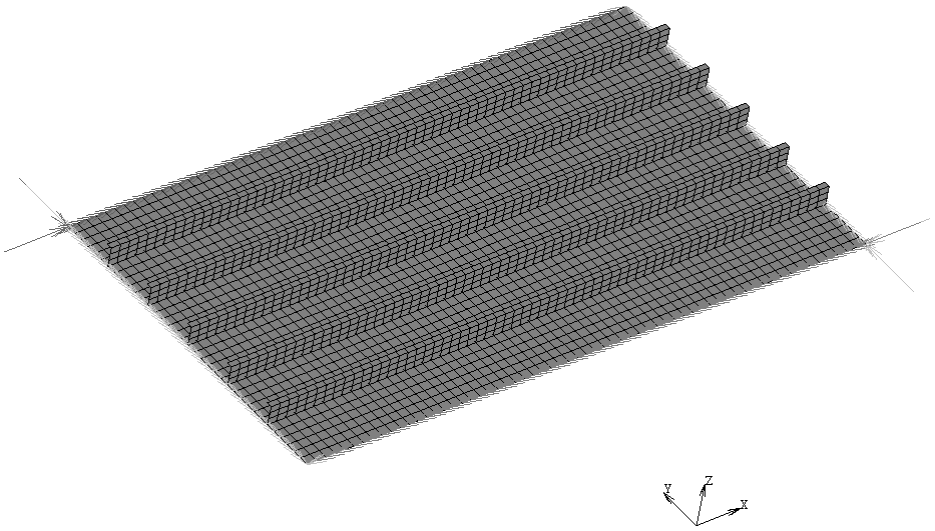


Figure 5.4.2 (3) Application of Shear Force

(5) Lateral pressure

The effect of lateral pressure is to be considered as this pressure may lead to the bending of the plate panel and thus longitudinal compressive stress in plane or at the flange of stiffened panel.

In the course of loading, the lateral pressure is applied to the plate panel as a surface load, and the situation that on which side the pressure is applied should be carefully considered. If the actual side of the stiffened panel on which the pressure applies is not clear, both situations when the pressure applies on either sides of the stiffened panel are to be considered respectively.

5.5 Initial imperfection

5.5.1 General requirements

(1) Initial imperfection of structure is to be taken into account for non-linear analysis, so the shape of the initial imperfection of the considered model is to be identified before calculation;

(2) For the initial imperfection mode and tolerance amplitude defined in the Guidelines, the potential residual stress and initial deformation during ship construction are implicitly considered;

(3) The initial imperfection is to be set so that the deformation mode of the structure is as consistent as possible without jumping along with the increase of load. If necessary, the initial imperfection is to be further reasonably identified in terms of shape and amplitude so that the initial imperfection set in the calculation is equal to or similar to the small imperfection necessary for the structure to fail in a reasonable collapse mode.

5.5.2 Imperfection model

It is to consider a global imperfection model by superposition of local imperfection shape components with overall imperfection components. The local components characterize the unevenness of plate panels and unevenness along the edge of stiffeners (plates between stiffeners, stiffener webs and stiffener flange plates). The overall imperfection is related to the stiffener unevenness measured perpendicularly to the plate panel.

5.5.3 Imperfection tolerance

The magnitude of the imperfection tolerance is to be taken as:

$$\delta_{P0} = s/200$$

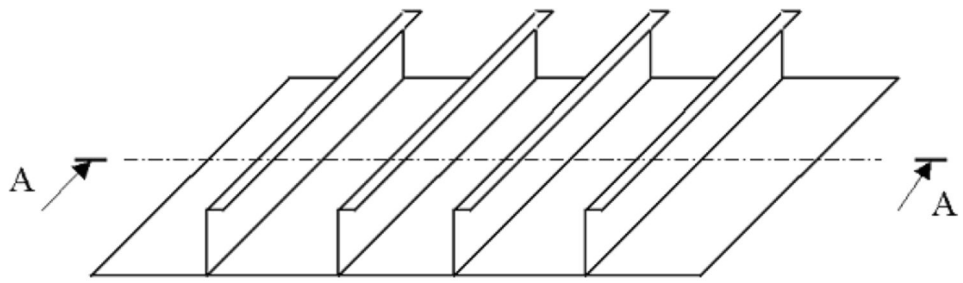
$$\delta_{S0} = l/1000$$

$$\delta_{T0} = l/1000$$

Where: l - span of longitudinal stiffeners, taken as the spacing between two adjacent transverse frames, in mm;

s - spacing between longitudinal stiffeners, in mm;

See Figure 5.5.3 for other symbols.



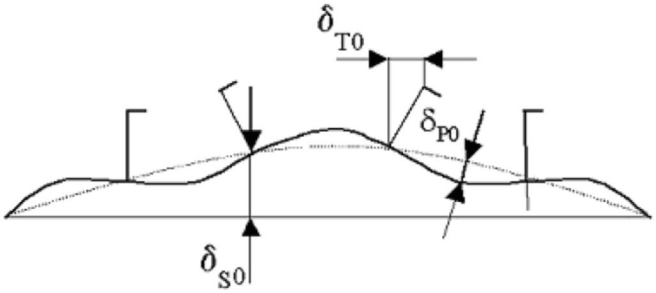


Figure 5.5.3 Imperfection Tolerance

5.5.4 Local imperfection mode

Generally, various load combinations in stiffened plate panels give rise to various buckling behaviors, so for non-linear collapse analysis, all potential initial imperfection modes are to be taken into account. The local imperfection modes to be analyzed are summarized in Table 5.5.4 for various load combinations.

Local Imperfection Modes to be Analyzed for Various Load Combinations Table 5.5.4

Load combination	Local imperfection mode
Pure longitudinal compression	- Buckling mode arising from pure longitudinal compression
Pure transverse compression	- Buckling mode arising from pure transverse compression
Bi-axial compression	- Buckling mode arising from pure longitudinal compression - Buckling mode arising from pure transverse compression
Bi-axial load (compression or tension) and/or shear stress	- Buckling mode arising from pure longitudinal compression - Buckling mode arising from pure transverse compression - Buckling mode arising from actual load combination

5.5.5 Overall imperfection mode

Generally, the overall imperfection for stiffened panel structure can be defined as:

(1) A half-sine wave along the length of the stiffener between primary supports (web frame/girder). See Figure 5.5.5 for the transverse direction.

(2) Imperfection of the stiffened panel structure at z axis is expressed by:

$$w_{S0} = \delta_{S0} \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

(3) Transverse tilting imperfection of stiffener is expressed by:

$$w_{T0} = -\delta_{T0} \frac{z}{h_w} \cos\left(\frac{\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right)$$

Where: δ_{S0}, δ_{S0} - the tolerance amplitude of stiffener imperfection given in 5.5.3;

h_w - height of stiffener web, in mm.



Figure 5.5.5 Overall Imperfection (magnifying 100 times)

5.5.6 Combination of imperfections

The corresponding components of local and overall imperfections are to be superimposed to derive a global imperfection model. See Figure 5.5.6. The maximum lateral imperfection amplitude of the plate between stiffeners is approximately $\delta_{P0} + \delta_{S0}$.

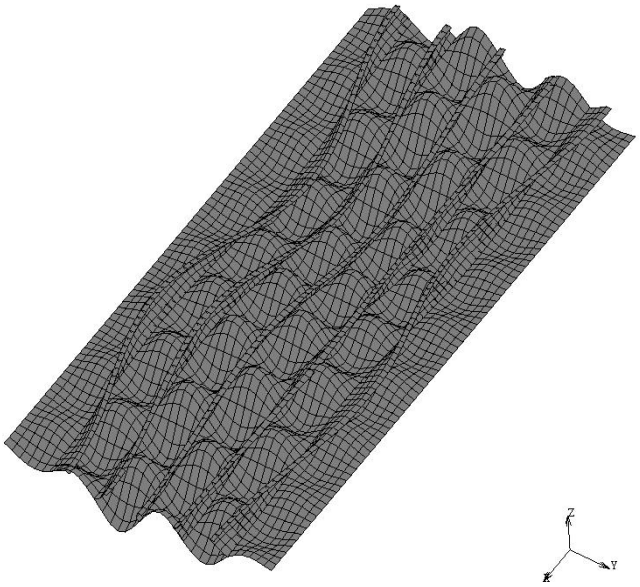


Figure 5.5.6 Combined global initial Imperfection (magnifying 100 times)

5.6 Solution procedure

5.6.1 Figure 5.6.1 shows the solution flow chart for non-linear finite element strength analysis of stiffened plate panels.

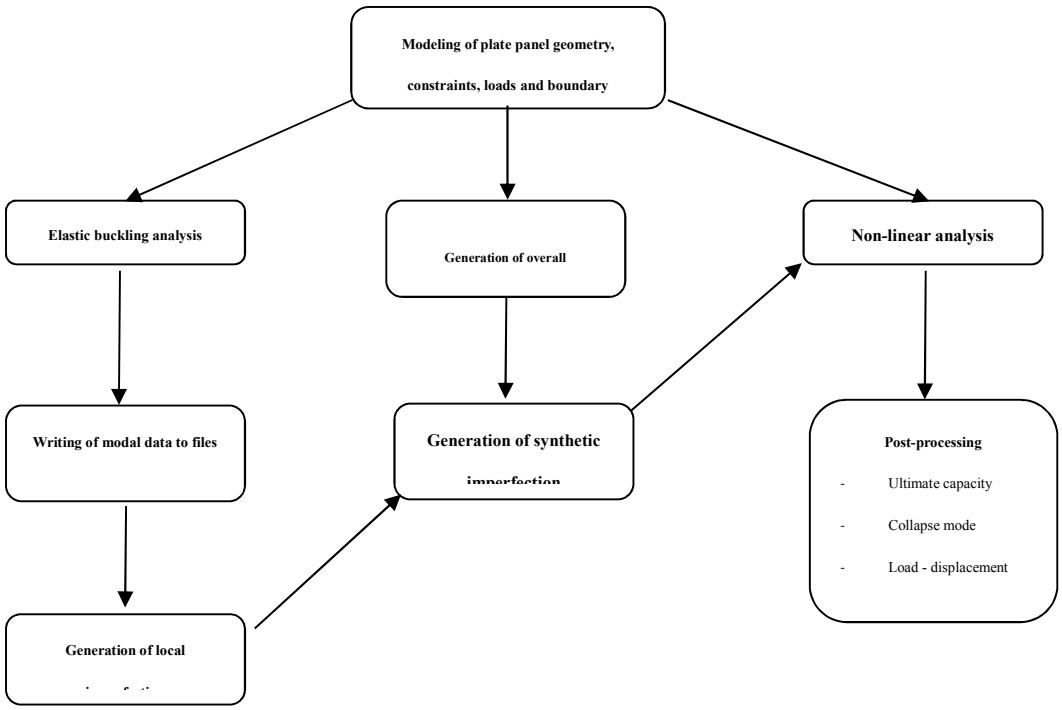


Figure 5.6.1 Flow Chart for Non-linear Finite Element Strength Analysis of Stiffened Plate Panels

5.7 NLFEM calculation software COMPASS-ABA-NLFEM

5.7.1 This software is a non-linear finite element calculation and analysis program developed by CCS according to the relevant requirements of this chapter for buckling and ultimate strength of stiffened panel structure.

5.7.2 This software can process the modelling, calculation and analysis automatically as far as possible for non-linear buckling analysis of stiffened panel structure, to avoid complex modelling and analysis work, including establishing detailed finite element analysis models, setting boundary conditions, calculating and applying initial imperfections, as well as applying boundary loads.

5.7.3 This software is subject to secondary development based on general purpose finite element analysis software. Its main interface is shown in Figure 5.7.3.

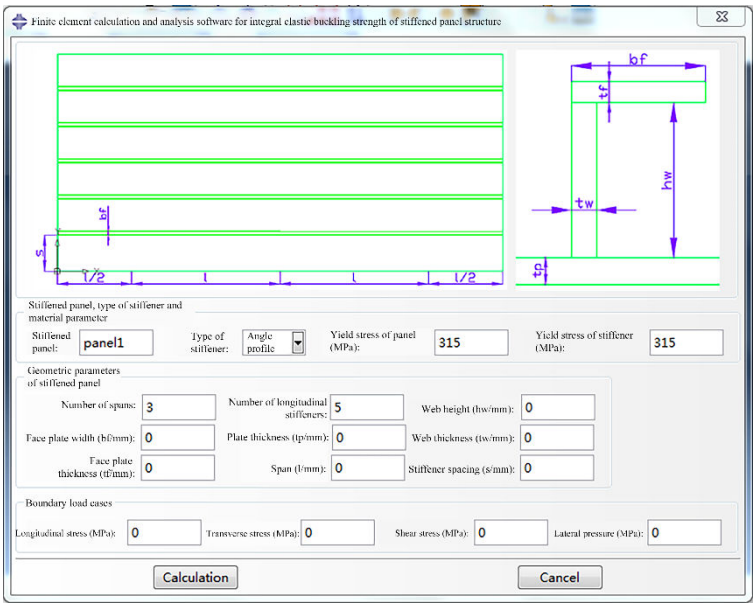


Figure 5.7.3 Main Interface of COMPASS-ABA-NLFEM Software

5.7.4 The functions below can be performed with this software:

(1) Realize the automation of modeling, and quickly establish a standardized and accurate finite element analysis model for stiffened panel structure.

(2) Elastic buckling strength analysis for local plate panels and overall stiffened panel structure.

(3) Non-linear finite element analysis for ultimate strength of stiffened panel structure.

5.7.5 The software allows for input of plate panel information and load cases via the interface, or import of the above input data from external files.

5.7.6 Detailed instructions on using the COMPASS-ABA-NLFEM software refer to its user manual.